A novel method for Rapid 3D fat and water decomposition using a GlObally Optimal multi-surface Estimation (R-GOOSE)

Chen Cui[1], Abhay Shah[1], Xiaodong Wu[1], Dan Thedens [2], and Mathews Jacob[1]

[1]*Electrical and Computer Engineering, University of Iowa, Iowa City, IA, United States,* [2]*Radiology, University of Iowa, Iowa City, IA, United States*

Synopsis

A 3D Rapid, GlObally Optimal Surface Estimation (R-GOOSE) algorithm for fat-water decomposition in MRI is proposed. The fat-water separation is formulated as an optimization problem with data consistency and field-map smoothness penalty. The data consistency only contains exact minimizers from the fully discretized field-map value volume. The proposed method employs a connectivity-reduced graph construction that enables the new formulation to be solved efficiently. The method is validated by the 17 datasets from the 2012 ISMRM Challenge with thirty-fold computational gain compared to our previous method GOOSE while the high quantitative accuracy is maintained. Fat fraction maps obtained from the proposed method also provides a good marker for degenerative muscle diseases in newly collected lower limb datasets.

Purpose

The estimation of fat and water from multi-echo MR images is a classical problem. The non-linearity of the image model and the ambiguities often make the estimation from few numbers of echoes challenging. Iterative algorithms that exploit the smoothness of field map[1,2,3,4] are able to overcome the problem in simpler situations. However, none of the above algorithms are guaranteed to converge to the global optimum of the cost function, leading to fat-water or phase wraps in challenging applications. A single-step graph search algorithm termed as GOOSE[5], guaranteed to converge to the global minimum, was recently introduced. While this scheme was able to provide considerably improved results over the state-of-the-art, this scheme had a high computational complexity, restricting its application in large-scale 3D problems. Here, we introduce a new algorithm, which inherits the global optimality of GOOSE, while reducing the computational complexity by an order of magnitude.

Methods

The fat-water signal at each pixel $\mathbf{r} = (x, y, z)$ can be mathematically expressed as a succession of images in different echo-times (ET):

$$\underbrace{\begin{bmatrix} s(\mathbf{r}, t_1) \\ \vdots \\ s(\mathbf{r}, t_N) \end{bmatrix}}_{\mathbf{s}(\mathbf{r})} = \underbrace{\begin{bmatrix} e^{-jf(\mathbf{r})t_1} & \left(\sum_{i=1}^{M} \beta_i \, e^{j2\pi\delta_i - f(\mathbf{r})\, t_n}\right) \\ \vdots & \vdots \\ e^{-jf(\mathbf{r})t_N} & \left(\sum_{i=1}^{M} \beta_i \, e^{j2\pi\delta_i - f(\mathbf{r})\, t_N}\right) \end{bmatrix}}_{\mathbf{A}_f(\mathbf{r})} \underbrace{\begin{bmatrix} \rho_{\text{water}}(\mathbf{r}) \\ \rho_{\text{fat}}(\mathbf{r}) \end{bmatrix}}_{\boldsymbol{\rho}(\mathbf{r})}$$

Classical methods aims to estimate the field $f(\mathbf{r})$ by minimizing the criterion $D\left(f(\mathbf{r})\right) = \min_{\rho} \|\mathbf{s}(\mathbf{r}) - \mathbf{A_f}(\mathbf{r})\boldsymbol{\rho}\|^2$. GOOSE discretized the field-map at each pixel value onto a uniform grid $R_f = n\Delta$, where Δ is the grid spacing. It made the problem well-posed using an implicit smoothness constraint, illustrated in Figure 1(c). The global optimum of the constrained optimization problem was determined using a graph search algorithm. But the main drawback was the high computational complexity, resulting from the large number of graph layers.

In this work, we introduce a novel formulation that restricts the graph search to the local minima of $D\left(f(\mathbf{r})\right)$ at each voxel. This results in far fewer graph layers, which considerably reduces the computational complexity. The algorithm still inherits the global optimality of GOOSE. The local minima of $\mathcal{D}(f(\mathbf{r}))$, denoted by the set $\mathrm{LM}(\mathbf{r})$, is determined using a fast discrete search. The field-map estimation is posed as the smoothness penalized 3D optimization problem:

$$\{\hat{f(\mathbf{r})}; \mathbf{r} \in G\} = \arg \min_{\{\hat{f(\mathbf{r})}; \mathbf{r} \in G\}} \sum_{\mathbf{r} \in G} [\mathcal{D}(f(\mathbf{r})) + \sum_{\mathbf{s} \in N(\mathbf{p})} \mu_{\mathbf{rs}} |f(\mathbf{r}) - f(\mathbf{s})|^2], \;\; f(\mathbf{r}) \in \mathrm{LM}(\mathbf{r})$$

Here, $N(\mathbf{r})$ is the local neighborhood of the voxel $\mathbf{r}$ and $\mu_{\mathbf{rs}}$ are predefined weights. The first term is the data consistency cost, while the second encourages field-map smoothness.

To solve the above formulation, we build a graph whose nodes at each pixel are the entries of $\mathrm{LM}(\mathbf{r})$. The node costs are specified by $\mathcal{D}(f(\mathbf{r}))$ and the smoothness costs are by $\mu_{\mathbf{rs}} |f(\mathbf{r}) - f(\mathbf{s})|^2$, which are distributed between edges. The graph problem is solved using the graph construction introduced in [7]. Once the optimal field map is obtained, the fat and water concentrations are estimated as $\rho = (\mathbf{A}_f^T \mathbf{A}_f)^{-1} \mathbf{A}_f^T \mathbf{s}$.

Results

We first validate the proposed algorithm using the ISMRM metric across 17 datasets from the 2012 ISMRM Challenge[7]. We observe that the proposed scheme on average takes 10 seconds/dataset and provides an average score of 99.34%. We also implemented a multi-scale version of the algorithm similar to [8] to further reduce the computation time. Specifically, the field-map solution obtained from a down-sampled dataset is propagated to the finer scale. This further reduced the run time to an average of 4.5 seconds, while score improved marginally to 99.42%. See Table 1 for details. In Figure 2, a challenging case (Data 12) due to its SNR and the isolated 'island' of object (water) is shown. The proposed method successfully identified the island as water and achieves a score of 97.63%.

We are currently using the algorithm to evaluate fat fraction map as a biomarker for disease progression and treatment efficacy in neurodegenerative muscle diseases such as myotonic dystrophy (DM1) and Charcot-Marie-Tooth disease (CMT), which are characterized by fatty infiltration and muscular atrophy. An example from the study on a bilateral lower limb dataset, acquired with a gradient-echo-based 3-point Dixon sequence on a GE 750w scanner is shown in Figure 3.

Acknowledgements

No acknowledgement found.

References

[1] Hernando D, Kellman P, Haldar JP, and Liang ZP. Robust water/fat separation in the presence of large field inhomogeneities using a graph cut algorithm. Magn Reson Med, 2010; 63:79–90.

[2] Berglund J, Johansson L, Ahlstrom H, and Kullberg J. Three-point Dixon method enables whole-body water and fat imaging of obese subjects. Magn Reson Med, 2010; 63:1659–1668.

[3] Lu W, Hargreaves BA. Multiresolution field map estimation using golden section search for water-fat separation. Magn Reson Med, 2008; 60:236–244.

[4] Tsao J and Jiang Y. Hierarchical IDEAL: Fast, robust, and multiresolution separation of multiple chem- ical species from multiple echo times. Magn Reson Med, 2012; 70:155–159.

[5] Cui C, Wu X, Newell JD, and Jacob M. Fat water decomposition using globally optimal surface estima- tion (goose) algorithm. Magn Reson Med, 2015; 73:1289–1299.

[6] Shay A, Bai J, Abramoff M, Wu X. Optimal Multiple Surface Segmentation with Convex Priors in Irregularly Sampled Space. https://arxiv.org/abs/1611.03059.

[7] http://challenge.ismrm.org/node/18

[8] Berglund J and Skorpil M. Multi-scale graph-cut algorithm for efficient water-fat separation. Magnetic Resonance in Medicine, 2016.

Figures

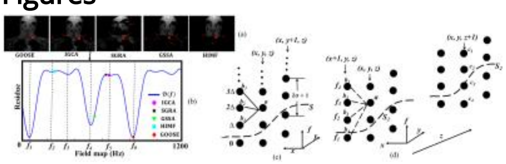

Figure 1: (a)-(b) Local and global convergences of different methods (IGCA[2], SGRA[3], GSSA[4], HIMF[5]) and the associated swaps in the fat image. (c) 3D optimal graph surfaces search algorithm model in GOOSE with hard constraint such as from a to b1, b2 and b3. (d) 4D graph space for graph search in the proposed method with the smoothness penalty for all edges.

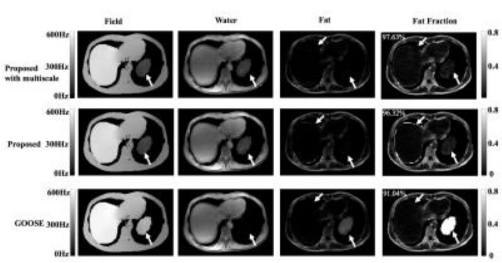

Figure 2: Field maps and fat-water separation results using both the proposed 3-D method, the proposed method with the multi-scaled scheme and GOOSE on the liver dataset.

Method	Mean	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Q(G)	99.27	99.84	99.81	96.50	99.87	99.94	99.88	99.90	99.94	99.97	99.72	99.75	95.58	99.91	99.87	99.15	99.13	98.80
Q(RG)	99.34	99.83	99.78	95.01	99.90	100	99.79	99.92	99.95	100	99.73	99.52	96.10	100	99.71	99.49	99.53	98.70
Q(mRG)	99.42	99.84	99.83	96.13	99.90	100	99.85	99.91	100	100	99.83	99.74	97.05	100	99.75	99.68	99.65	98.90
T(G)	323.9	220.7	183.5	269.2	319.1	318.8	160.8	314.4	700.9	829.3	224.9	536.7	522.4	192.4	422.7	76.7	154.8	59.4
T(RG)	10.7	9.7	9.4	7.4	9.0	17.1	11.1	13.3	12.4	19.2	13.5	14.2	15.2	2.1	13.6	8.1	3.2	3.6
T(mRG)	4.5	4.2	4.8	3.8	5.1	5.5	5.2	5.8	3.5	4.6	4.7	5.6	4.9	2.9	5.0	4.8	3.2	3.1

Table 1: Comparison of score and time between GOOSE (G), the proposed method(RG) and the the proposed method with the multi-scaled scheme (mRG) across the 17 datasets from the 2012 ISMRM Challenge.

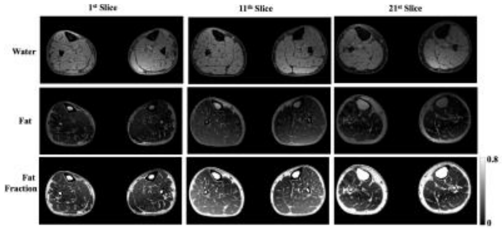

Figure 3: Field maps and fat-water separation results at 1st, 11th, and 21st slices of the bilateral lower limb dataset using the proposed 3D method.