MRI & manifolds

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Declaration of Financial Interests or Relationships

Speaker Name: Mathews Jacob

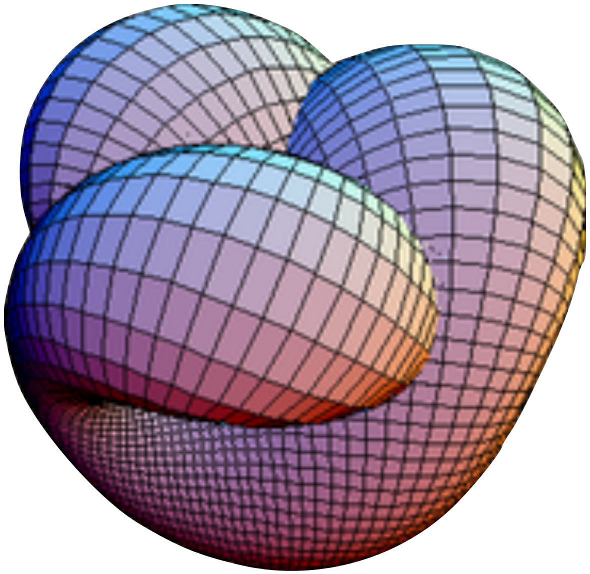
I have no financial interests or relationships to disclose with regard to the subject matter of this presentation.

What are manifolds ?

Topological space: locally resembles Euclidean space

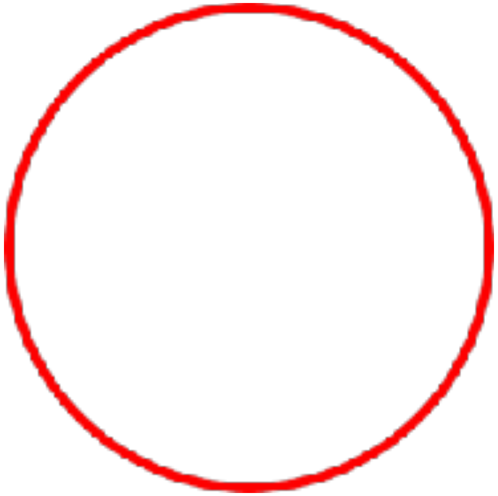
2-D manifold

n=2; N=3



2-D manifold

n=2; N=3



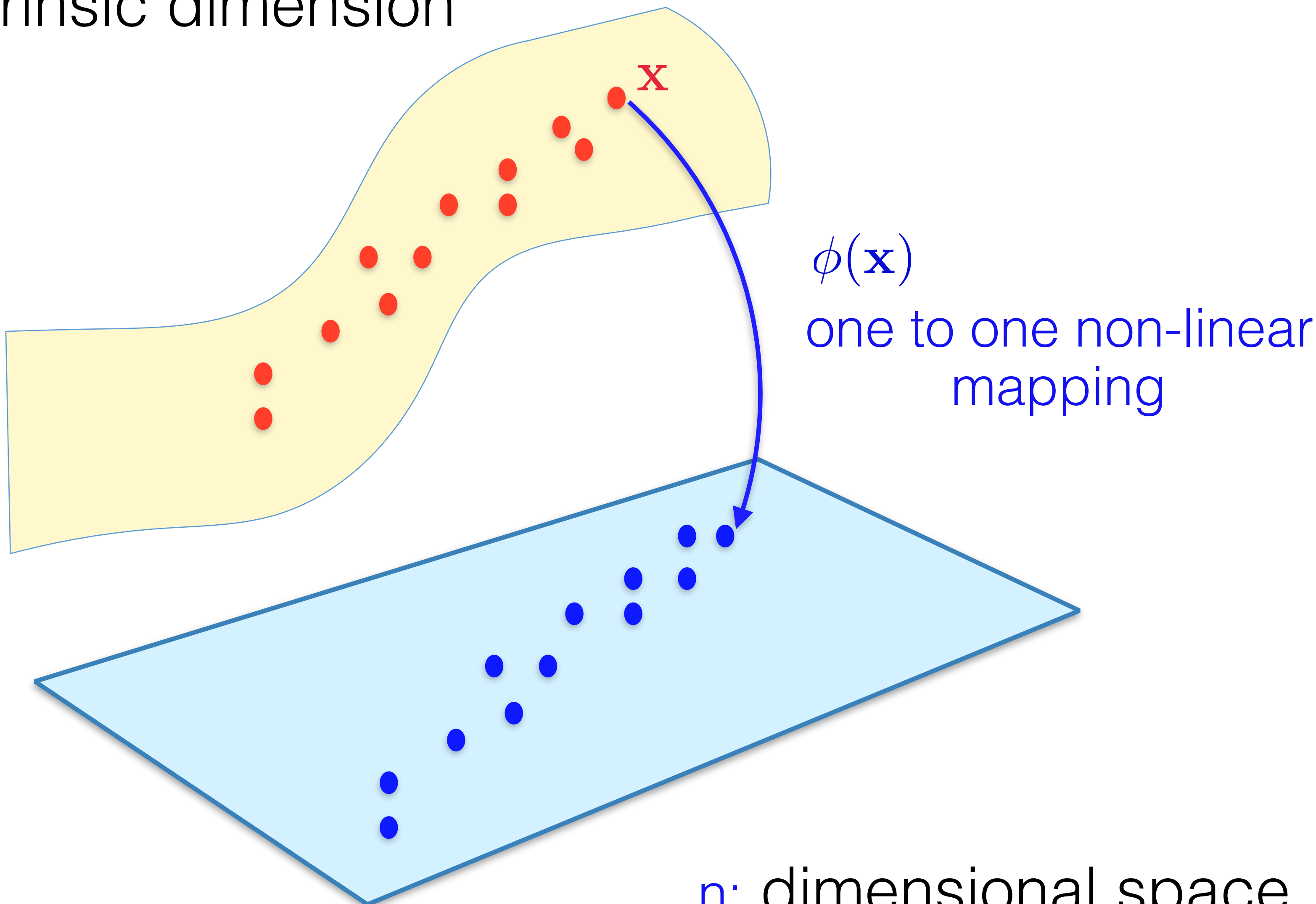
1-D manifold

n=1; N=2

n: dimension of the manifold
N: intrinsic dimension

Non-linear one to one mapping

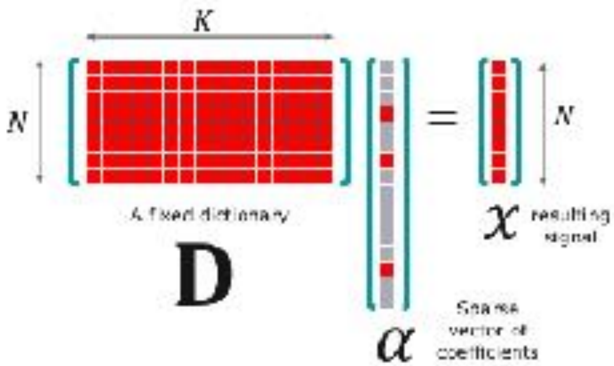
N: intrinsic dimension



n: dimensional space

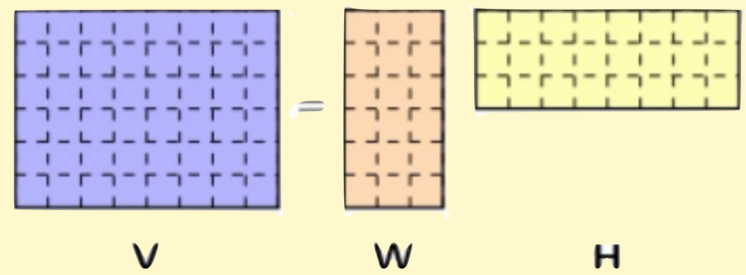
Current manifold models in MRI

Compressed sensing



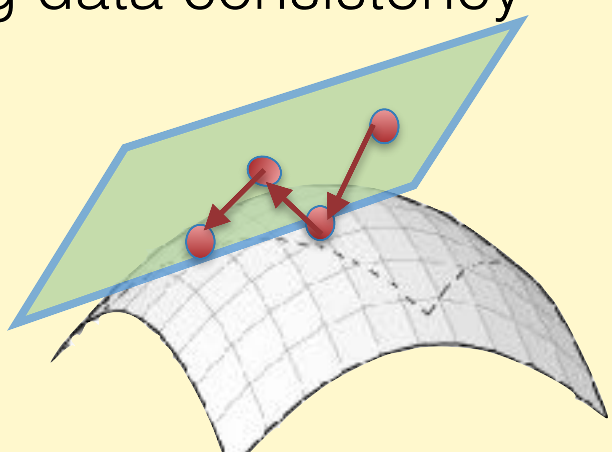
2k dimensional manifold

Low-rank models

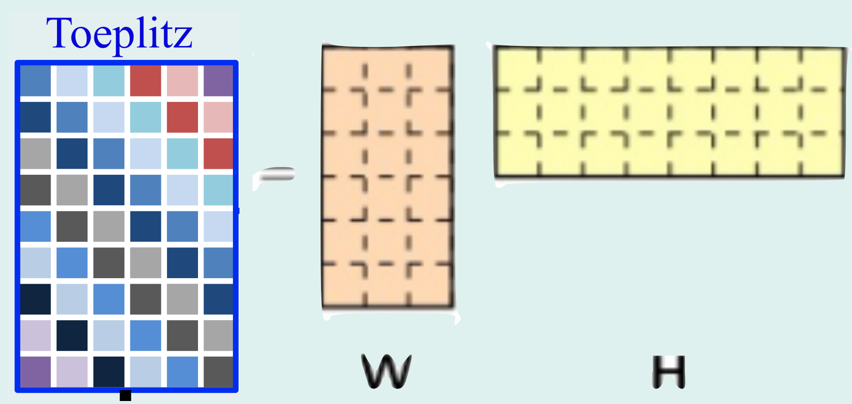


Grassman manifold
r(m+n-2r) dimensional

Points on manifold
Satisfying data consistency



Structured low-rank models



Manifold models in MRI

Matrix decomposition

Low rank and sparse models [Liang et al, Lingala & Jacob, ...]

Dictionary learning methods [Ravishankar et al, Lingala & Jacob,.....]

Structured low-rank matrix completion

Correlation in Fourier space [Ongie & Jacob, Haldar et al, Ye et al]

Smooth manifold models

Patch manifold: motion compensation [Yang & Jacob, Mohsin et al, ...

Image manifold: motion resolution [Poddar & Jacob, Nakarmi & Ying]

Manifold models in MRI

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Smooth manifold models

Patch manifold: motion resolution [Yang & Jacob, Mohsin et al, …..]

Image manifold: motion resolution [Poddar & Jacob, Nakarmi & Ying]

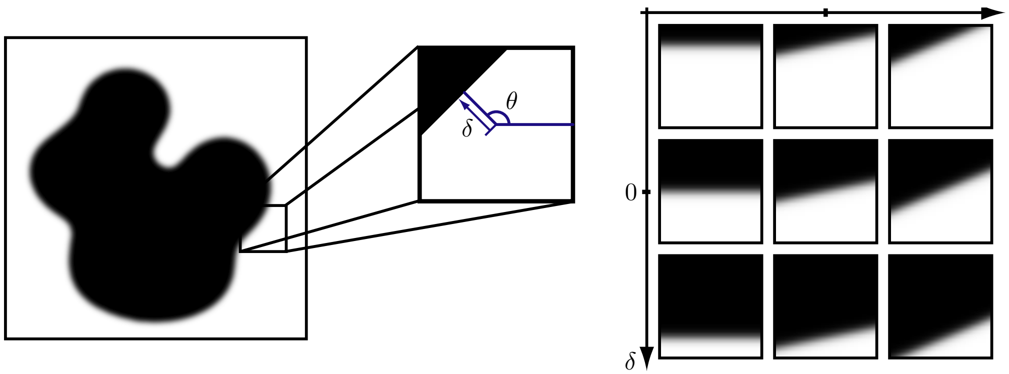
Smooth manifolds: examples

Patches in cartoon images

Non-linear function of

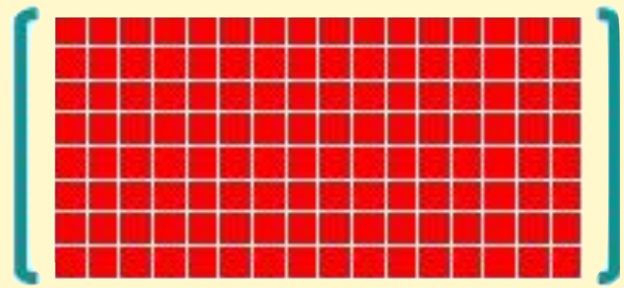
Orientation

Distance from origin



Gabriel Peyre, CVIU

Patch matrix



Rows: NL functions of

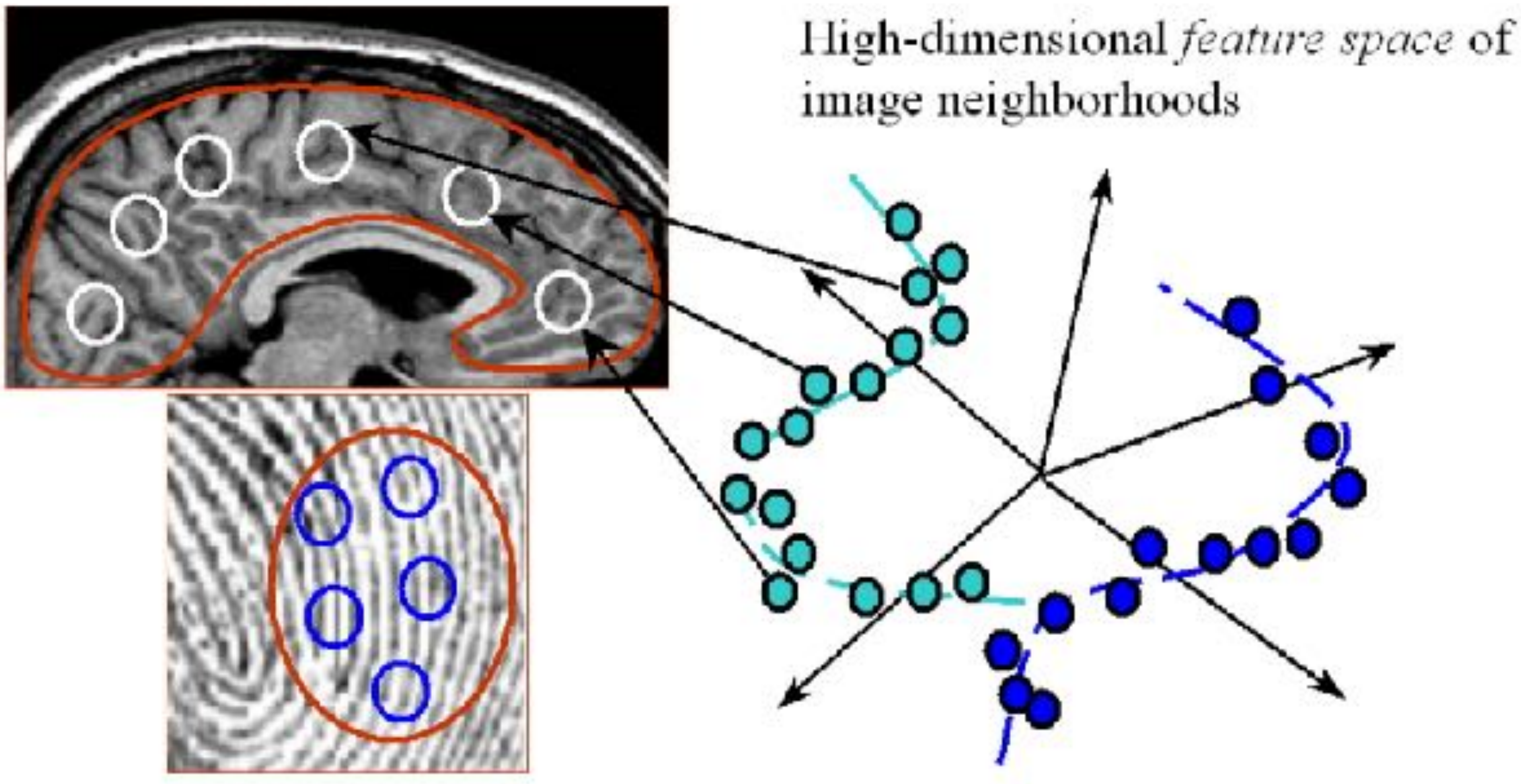
Orientation

Distance from origin

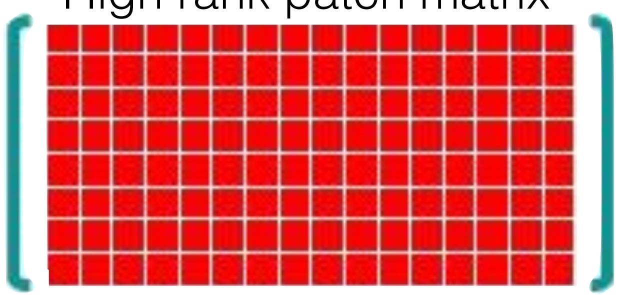
High rank matrix

Non-smooth manifold

Patch manifolds in natural images

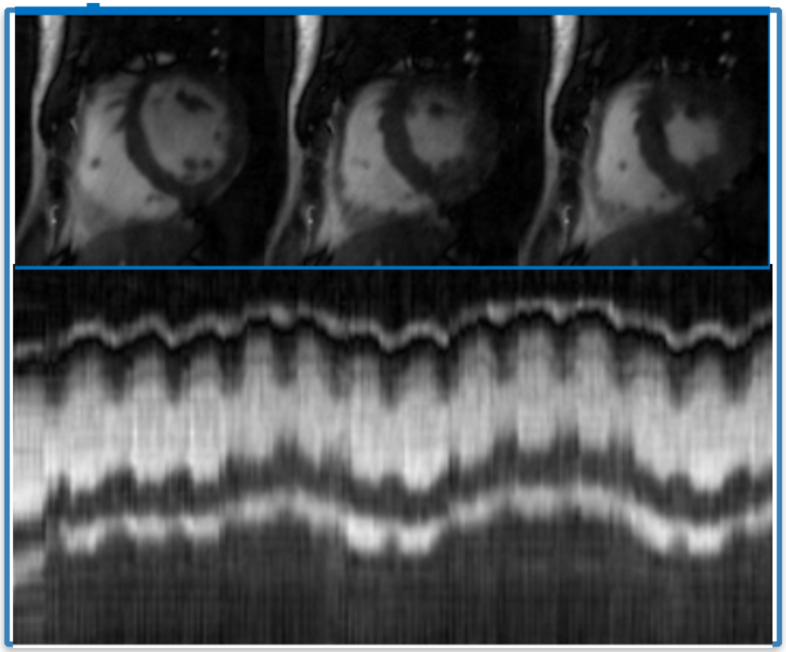


High rank patch matrix

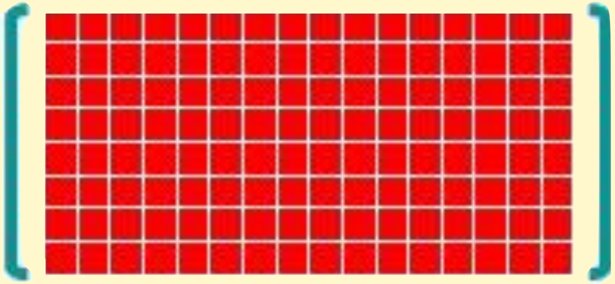


Awate et al, UINTA, TPAMI 2006

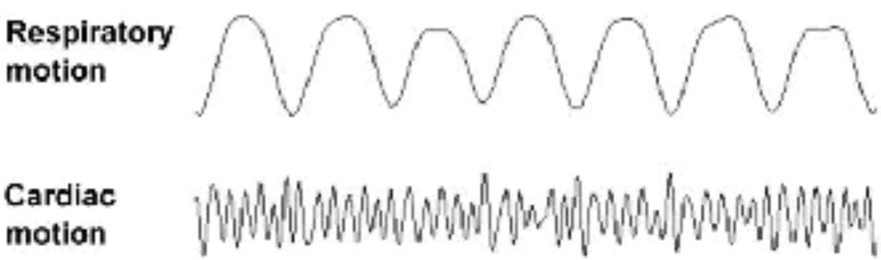
Image manifolds: FB CINE



High rank Casorati matrix

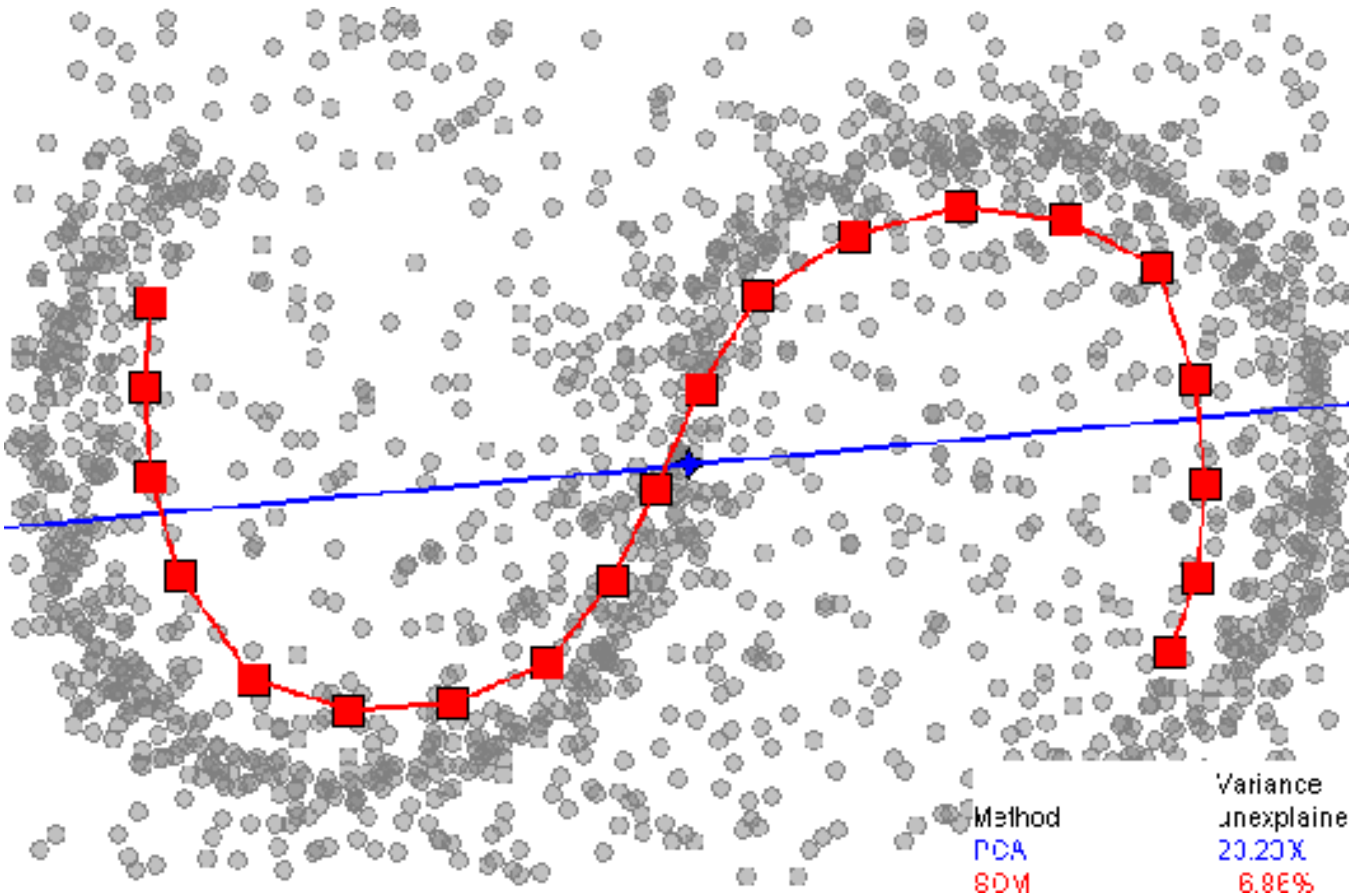


Low-rank priors not efficient



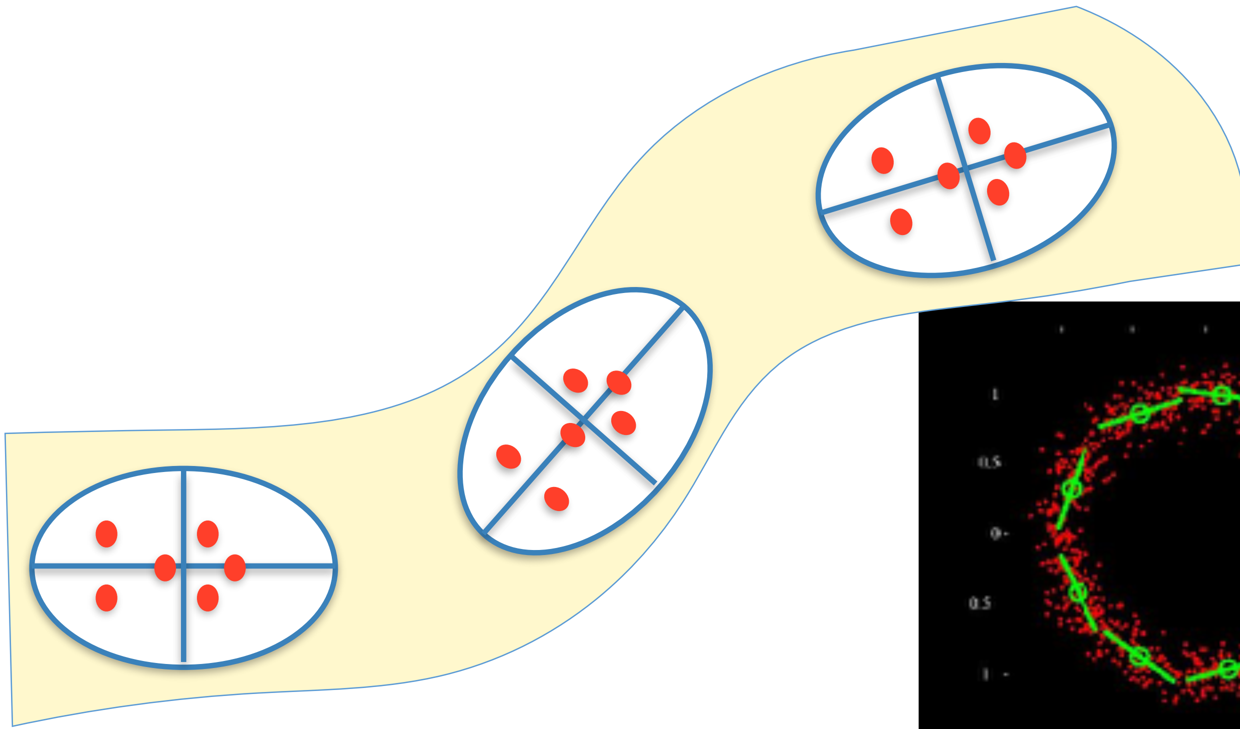
Images: smooth NL functions of cardiac time series

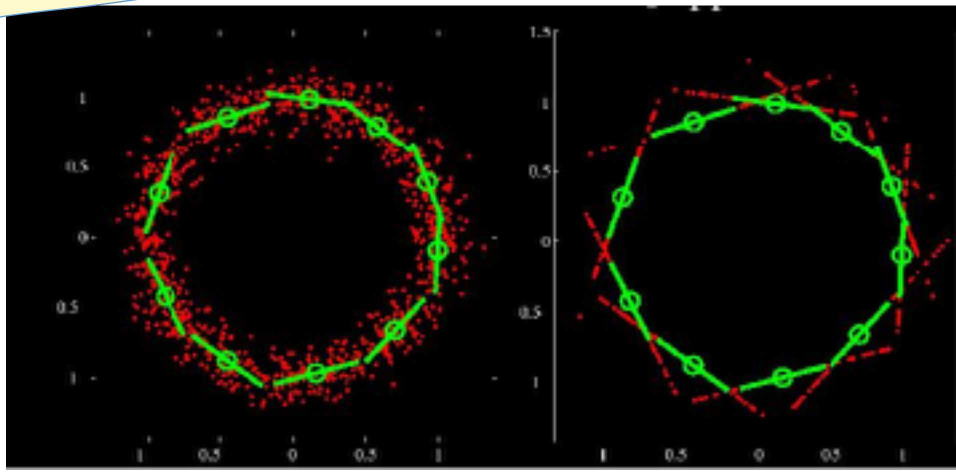
Low-rank approximation



Global linear model: inefficient in capturing manifold

Current methods with manifolds





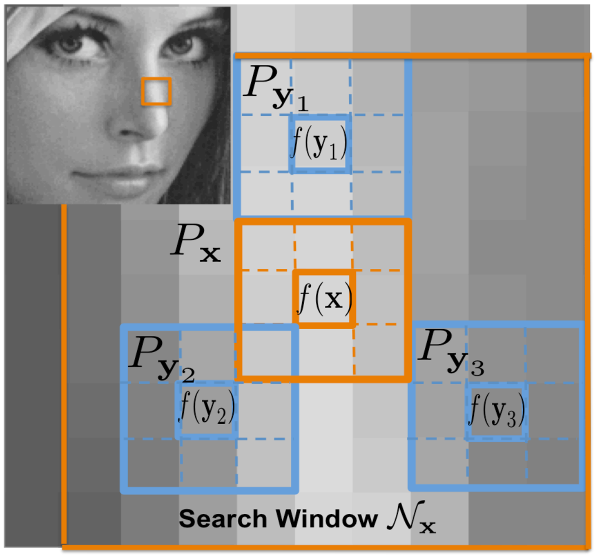
Model local neighborhoods using linear models

Dictionary learning/BCS

Mixture of PCA/ factor analysis

Non-local smoothing

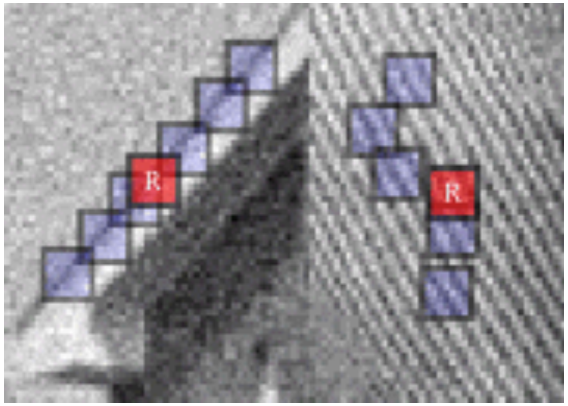
Unweighted sum of distances between patches



$$f_{\text{denoised}} = \frac{\sum_{\mathbf{y}\in\mathcal{N}_x} w(\mathbf{x},\mathbf{y}) f(\mathbf{y})}{\sum_{\mathbf{y}\in\mathcal{N}_x} w(\mathbf{x},\mathbf{y})}$$

$$w_{\mathbf{x},\mathbf{y}} = \exp\left(-\frac{\|\mathcal{P}_{\mathbf{x}}(f) - \mathcal{P}_{\mathbf{y}}(f)\|^2}{\sigma^2}\right)$$

Clusters patches to groups & average [BM3D]



Variational approach

The University of Iowa

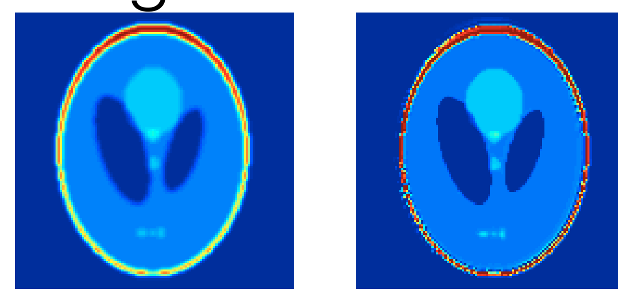
Regularized formulation for deblurring [Zhang et al, Lou et al]

$$f^* = \arg\min_{f} \|\mathcal{A}(f) - \mathbf{b}\|^2 + \lambda\, J_{\mathrm{NL}}(f)$$

$$J_{\mathrm{NL}}(f) = \sum_{\mathbf{r},\mathbf{s}} w\,[\mathbf{r},\mathbf{s}]\, \|f(\mathbf{r}) - f(\mathbf{s})\|^2$$

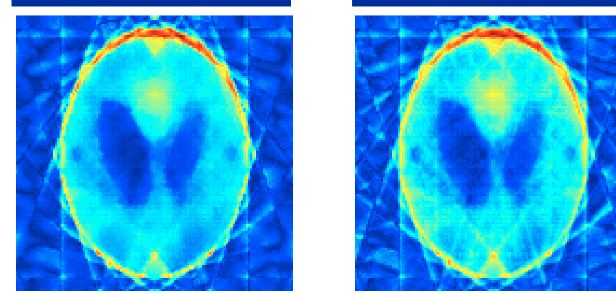
Weights estimated from blurred/noisy images

Works well for denoising/deblurring

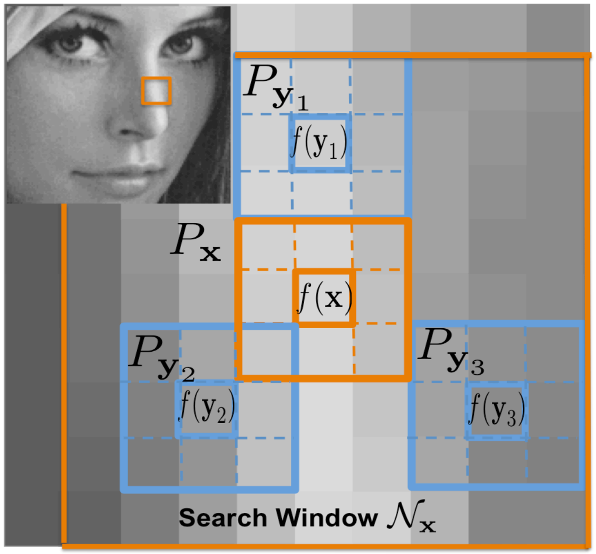


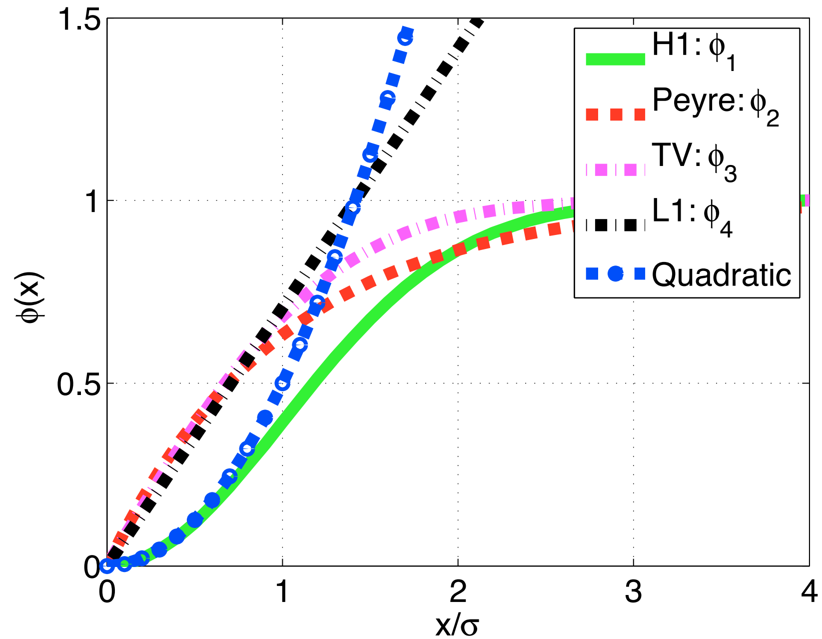
CS applications

IFFT weights result in poor estimates



Robust NL regularization





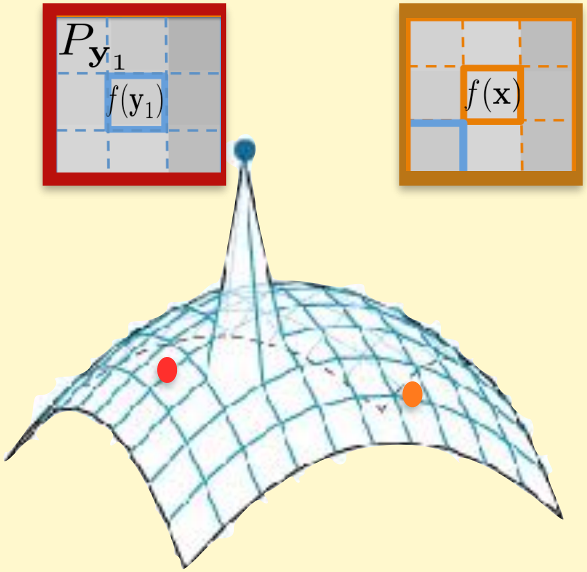
$$J_{\mathrm{NL}}(f) = \sum_{\mathbf{x},\mathbf{y}} \varphi\left(\|P_r(f) - P_s(f)\|\right)$$

Energy does not depend on weights

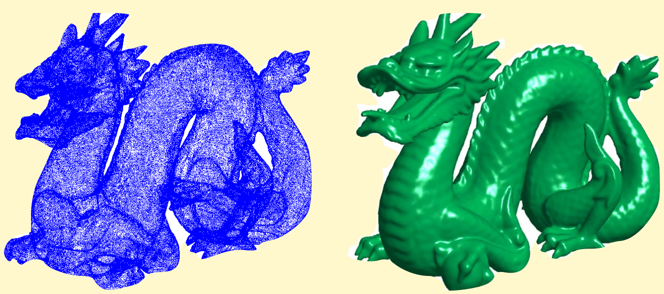
Applicable to general inverse problems

Yang & Jacob., IEEE TIP 2013

Why does this work ?



Denoising of manifolds



Denoising of point clouds

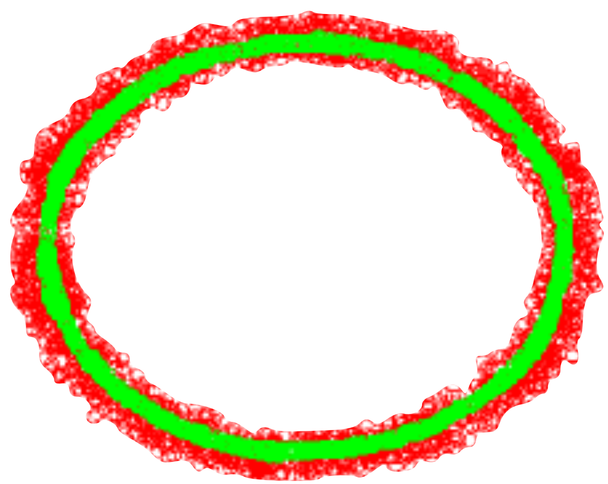
Smoothing of point clouds

Surface area of the mesh

$$\text{Area} \approx \sum_{\mathbf{x}} \sum_{\mathbf{y} \in N(\mathbf{x})} \psi\left(\|\mathbf{x}-\mathbf{y}\|\right)$$

Saturating distance: select neighborhood

Area minimization: curvature flow



Similarity to current methods

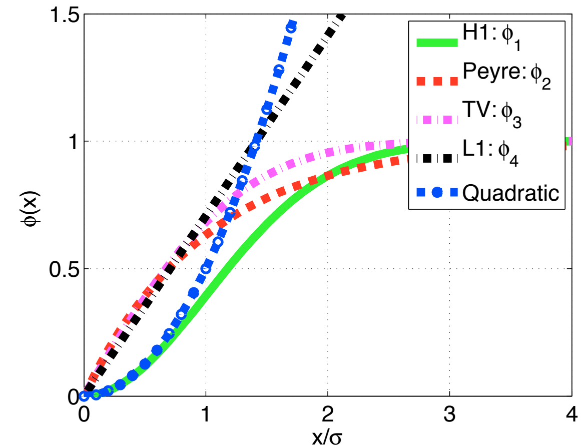
Majorization by a quadratic

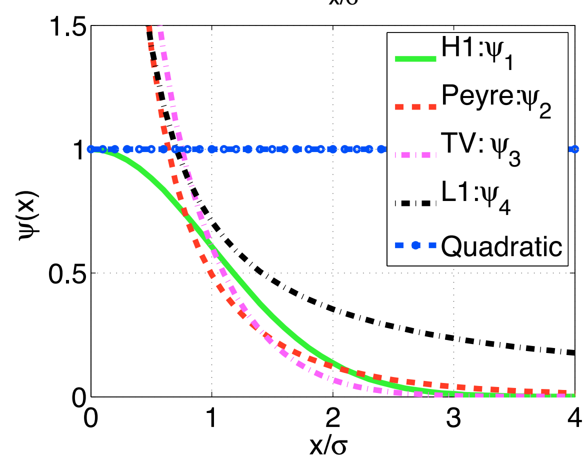
$$\varphi\left(\|P_{\mathbf{r}}(f) - P_{\mathbf{s}}(f)\|\right) \;\leq\; w_f(\mathbf{r},\mathbf{s})\,\|P_{\mathbf{r}}(f) - P_{\mathbf{s}}(f)\|^2 + c$$

Weights depend on patch differences

$$w_f(\mathbf{r},\mathbf{s}) = \psi(\|P_{\mathbf{r}}(f) - P_{\mathbf{s}}(f)\|^2)$$

$$\psi(x) = \frac{\varphi'(x)}{2\,x}$$





Non-local denoising methods

Majorization of the robust penalty

Iterative reweighted minimization

Alternate between manifold estimation and smoothing

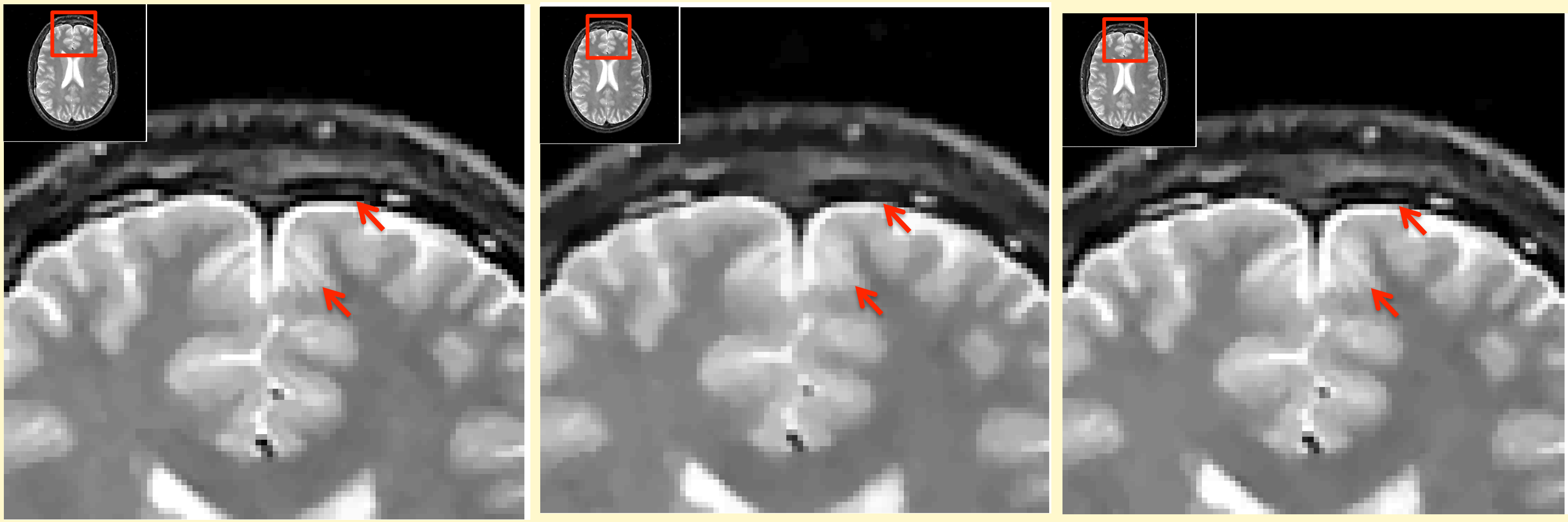
Manifold estimation

$$w_{n+1}[\mathbf{r},\mathbf{s}] = \psi\left(\|P_{\mathbf{r}}(f_n) - P_{\mathbf{s}}(f_n)\|^2\right)$$

Manifold smoothing

$$f_{n+1} = \arg\min_f \|\mathcal{A}(f) - \mathbf{b}\|^2 + \lambda \sum_{\mathbf{r},\mathbf{s}} w_{n+1}[\mathbf{r},\mathbf{s}]\, \|f(\mathbf{r}) - f(\mathbf{s})\|^2$$

Better performance @ runtime of TV



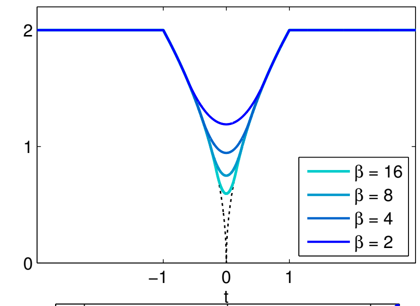
(f) Original image (a) Local TV, SNR=23.87 dB (d) NL-TV metric, SNR=28.09 dB

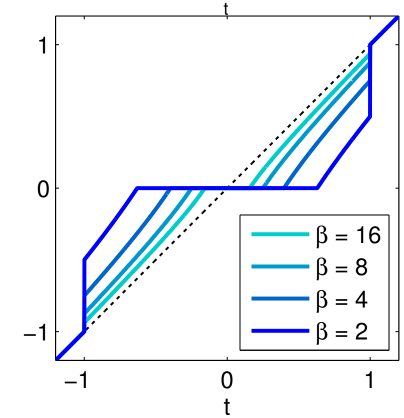
Yang & Jacob., IEEE TIP 2013

Iterative shrinkage algorithm

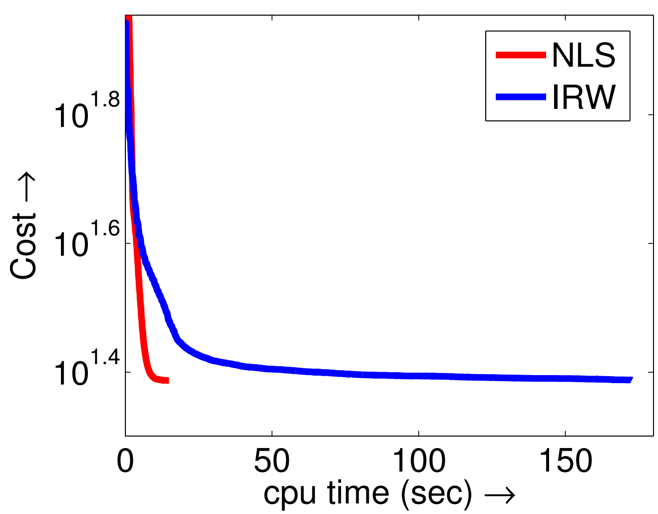
$$f^* = \arg\min_f \|\mathcal{A}(f) - \mathbf{b}\|^2 + \lambda\, J_{\mathrm{NL}}(f)$$

$$J_{\mathrm{NL}}(f) = \sum_{\mathbf{x},\mathbf{y}} \varphi\left(\|P_r(f) - P_s(f)\|\right)$$





Significantly faster convergence



Mohsin & Jacob., IEEE TMI 2015

Non-convex metrics: continuation

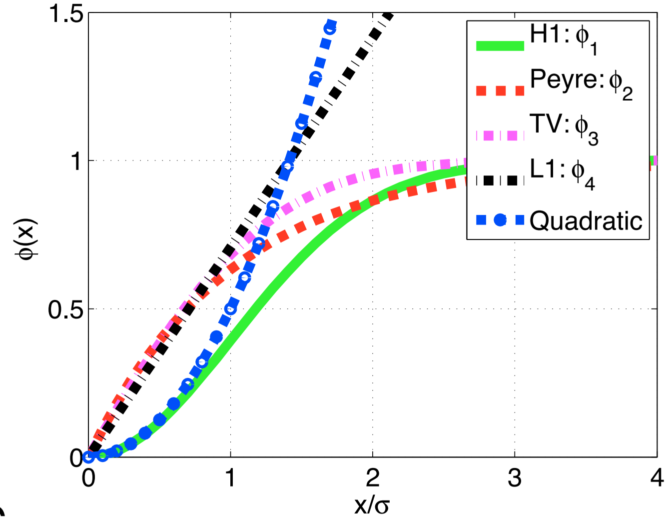
Non-convex metrics

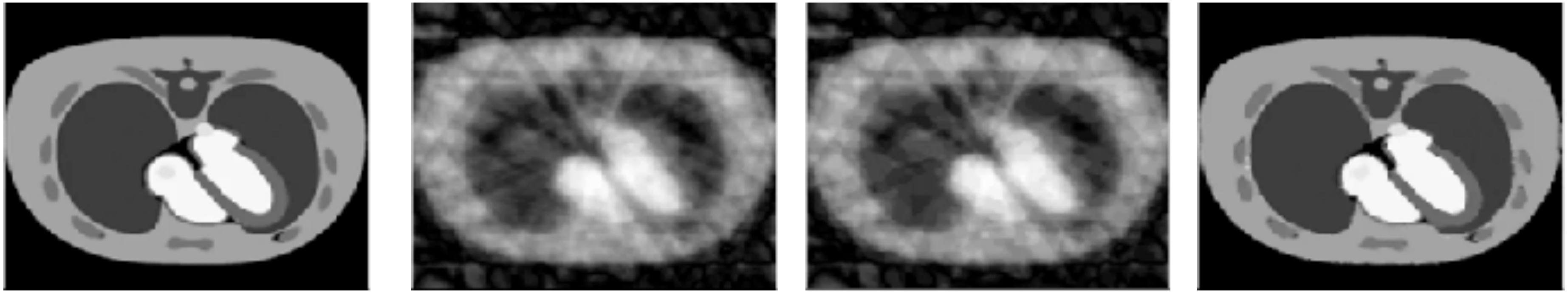
Convergence to local minima

Continuation strategy

Start with convex metrics

Gradually evolve to desired metric





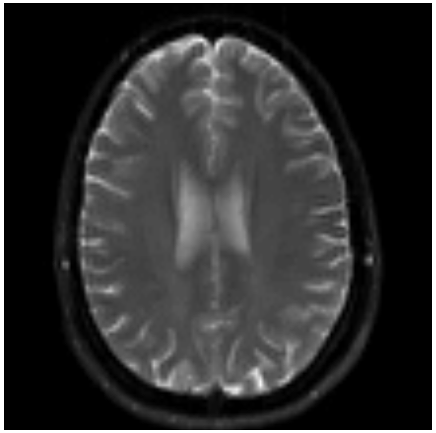
Original

Initial Guess

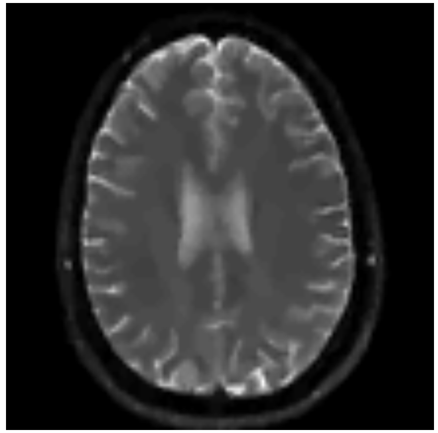
NLTV: without continuation

NLTV: with continuation

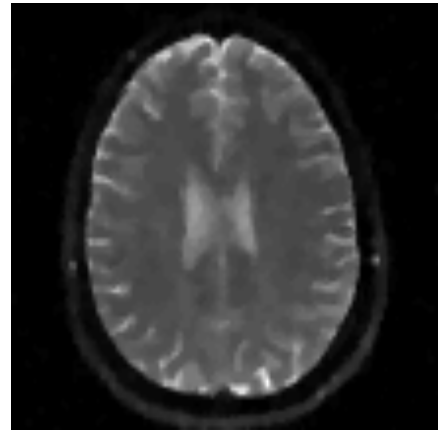
Comparison with state of the art



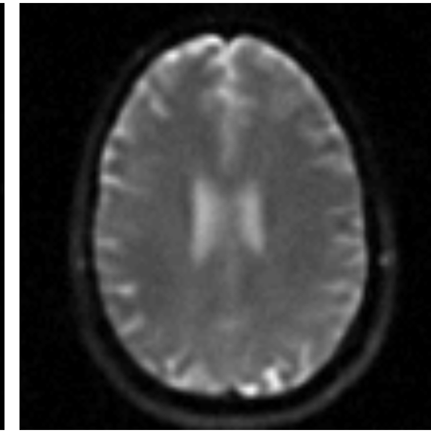
(a) Original image



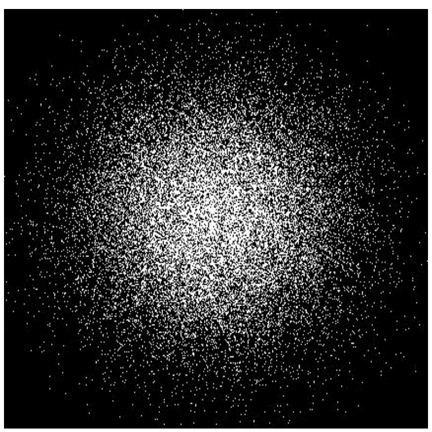
(b) NLS, SNR=23.3dB



(c) TV, SNR=19.2dB

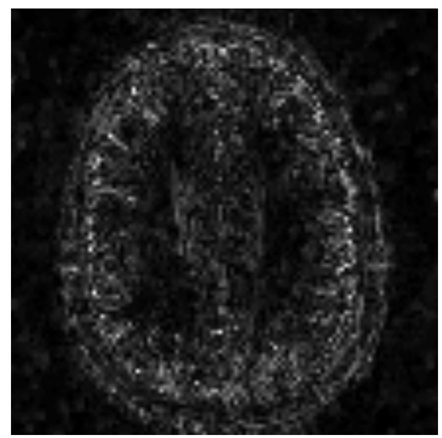


(d) DLMRI, SNR=16.6dB

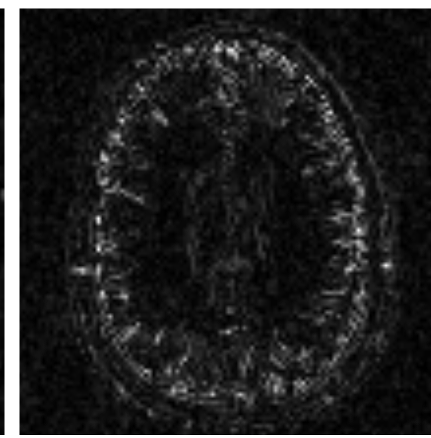


(e) Sampling pattern

(f) NLS error



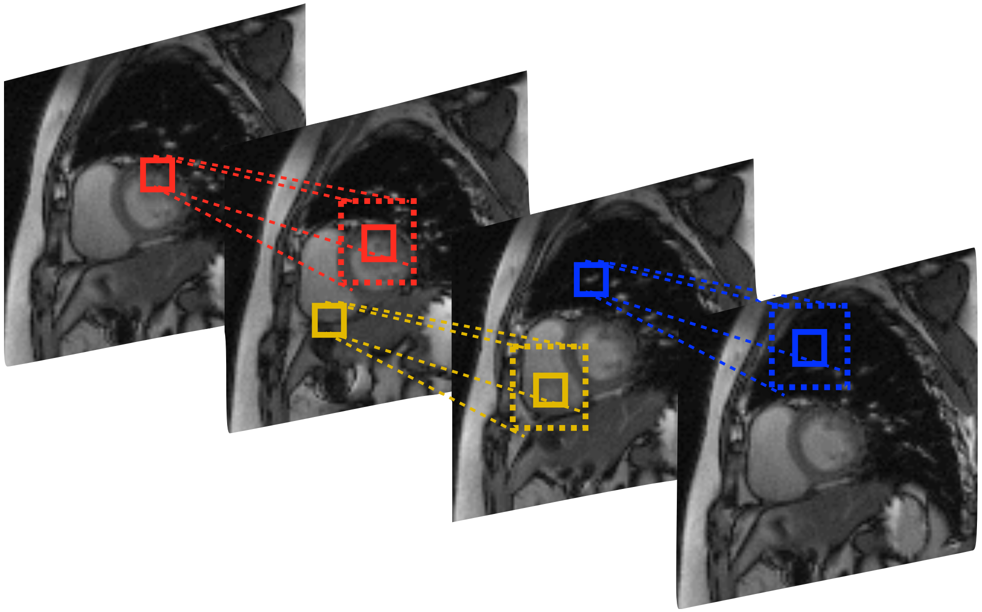
(g) TV error

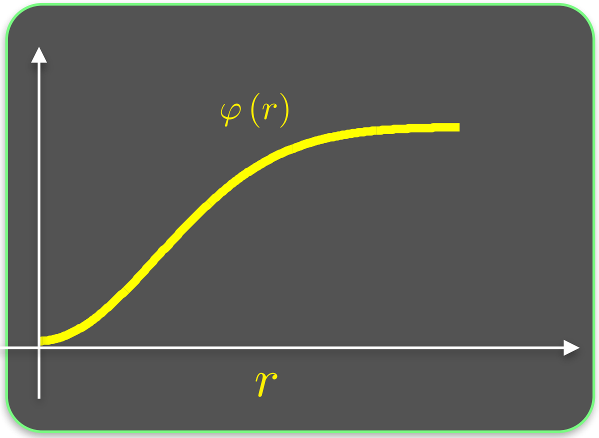


(h) DLMRI error

Mohsin & Jacob., IEEE TMI 2015

Implicit motion compensation



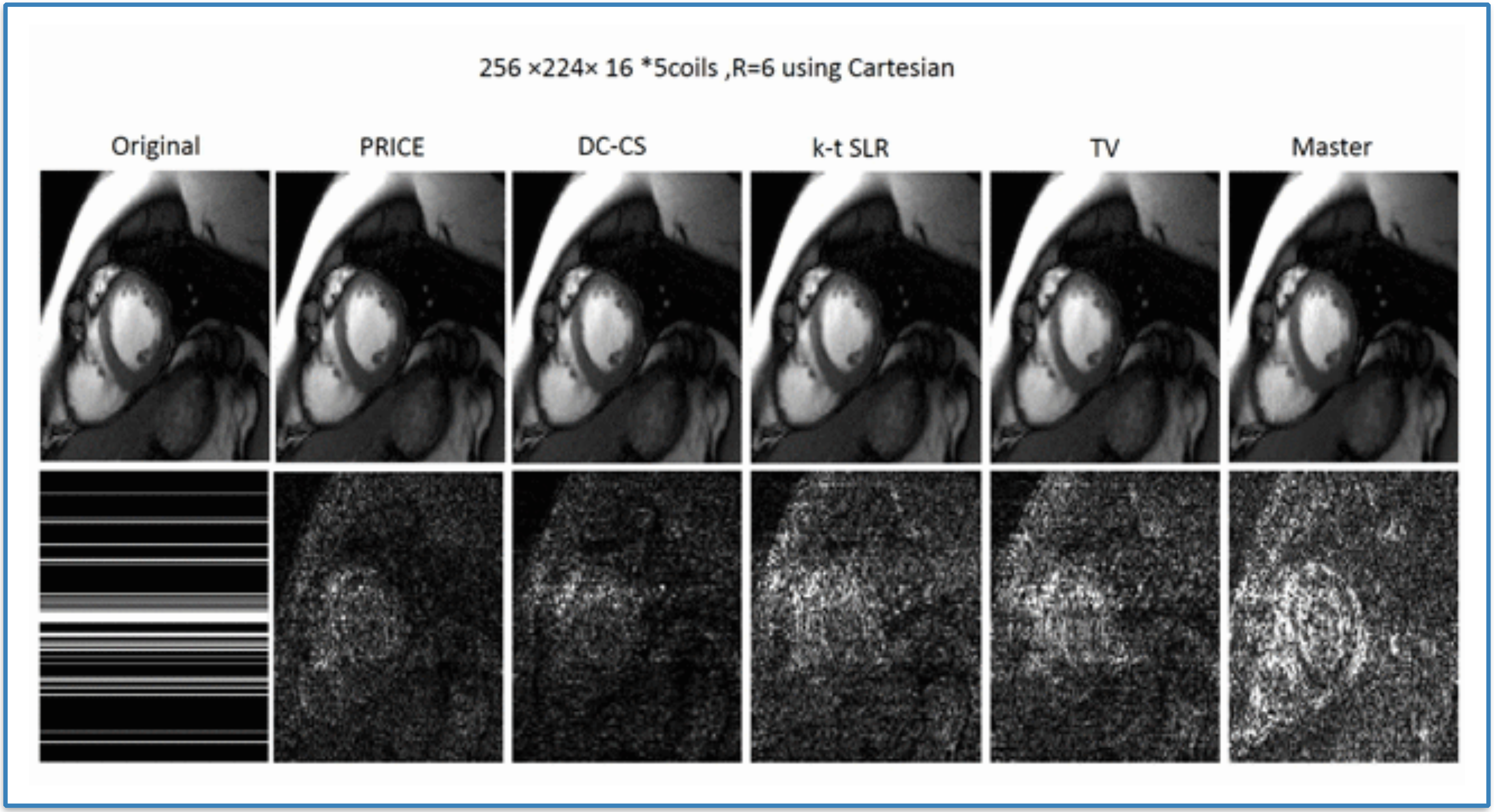


$$f = \arg\min_f \underbrace{\|\mathcal{A}(f) - b\|^2}_{\text{data consistency}} + \lambda \sum_{x,y \in \mathcal{N}_x} \varphi\left(\|P_x(f) - P_y(f)\|\right)$$

Minimize averaging of dissimilar patches

Complexity comparable to TV

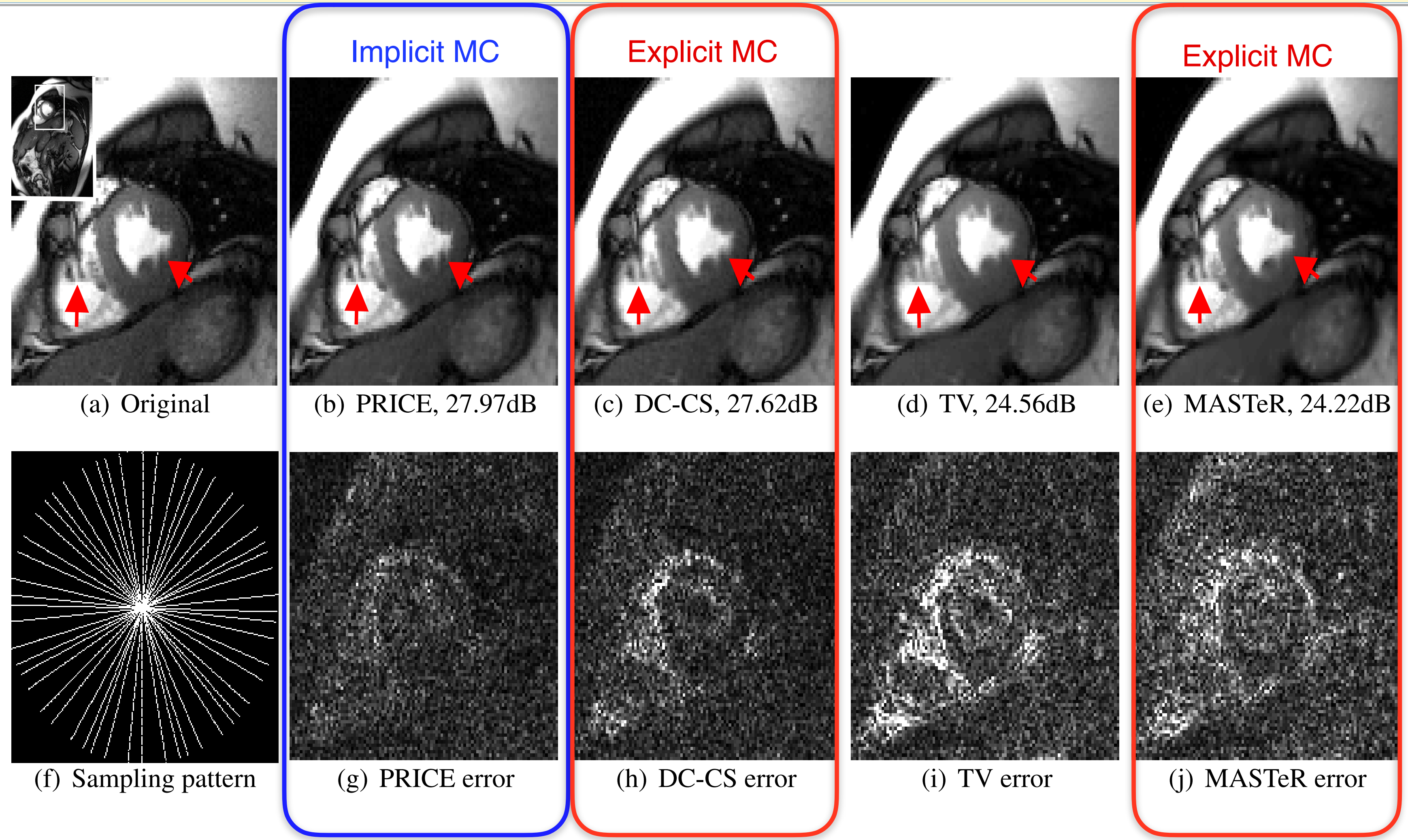
Cartesian CINE



Mohsin, Lingala, Dibella & Jacob., MRM 16

Software: https://research.engineering.uiowa.edu/cbig/content/price

Cartesian CINE



Mohsin, Lingala, Dibella & Jacob., MRM 16

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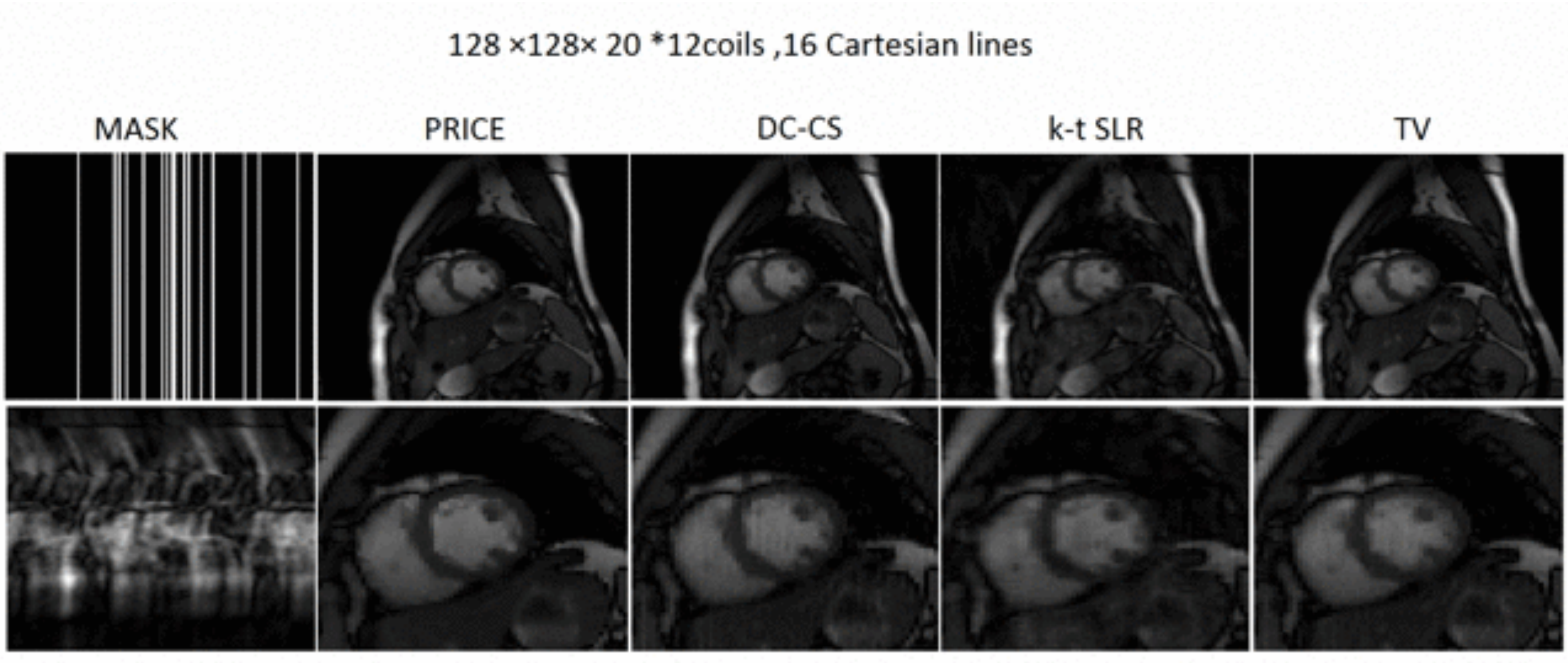
Comparisons with explicit MC



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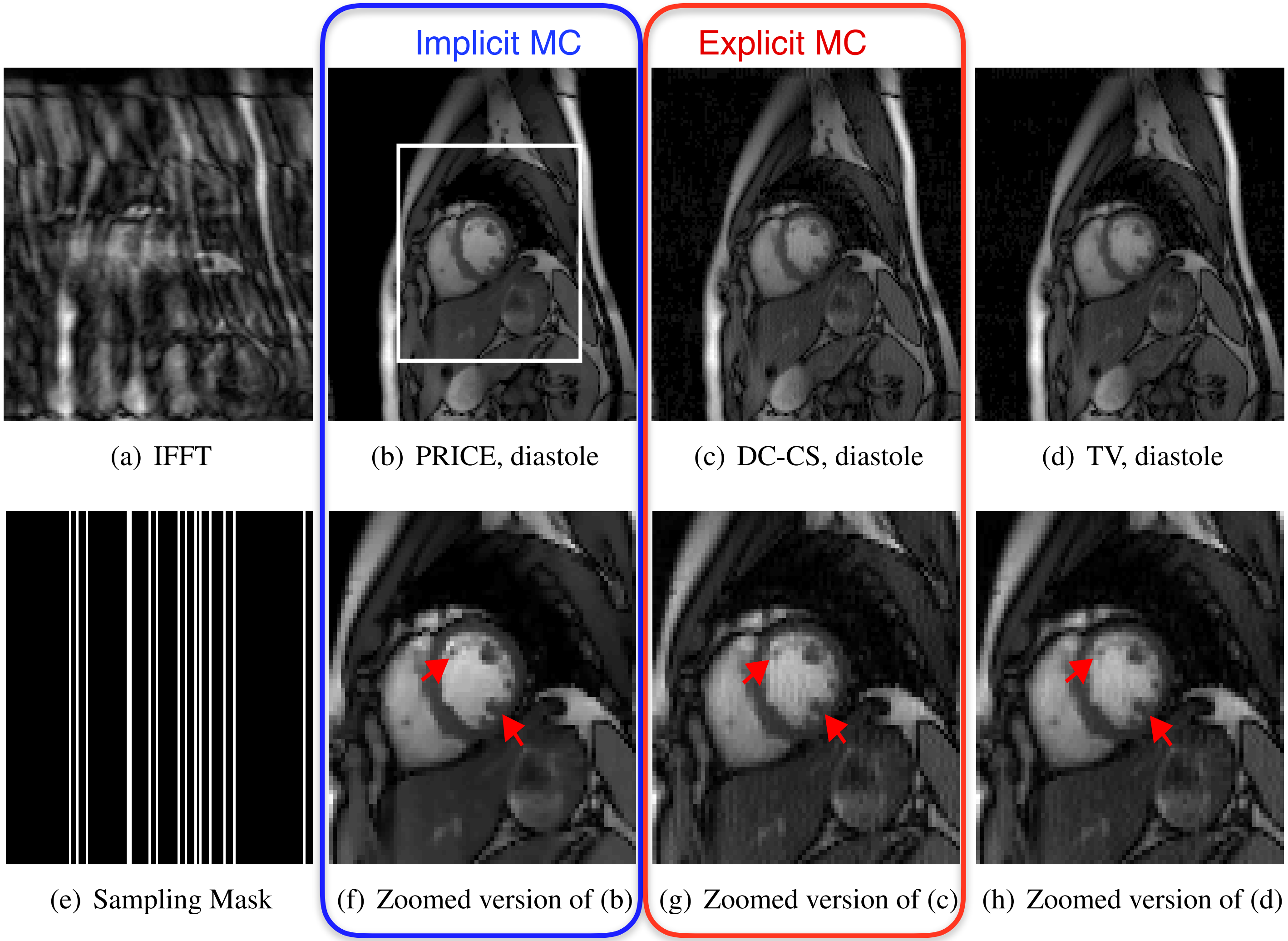

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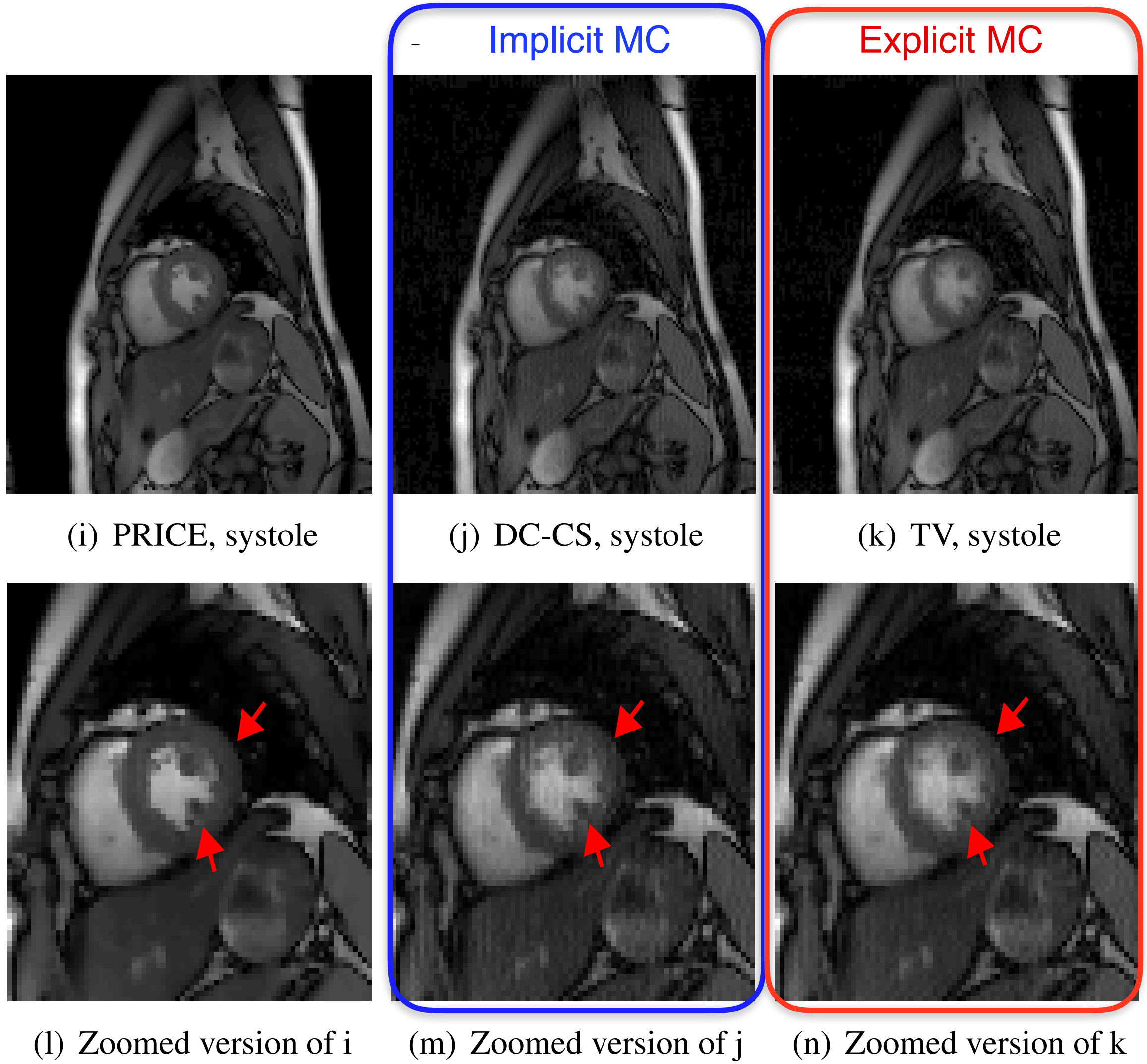

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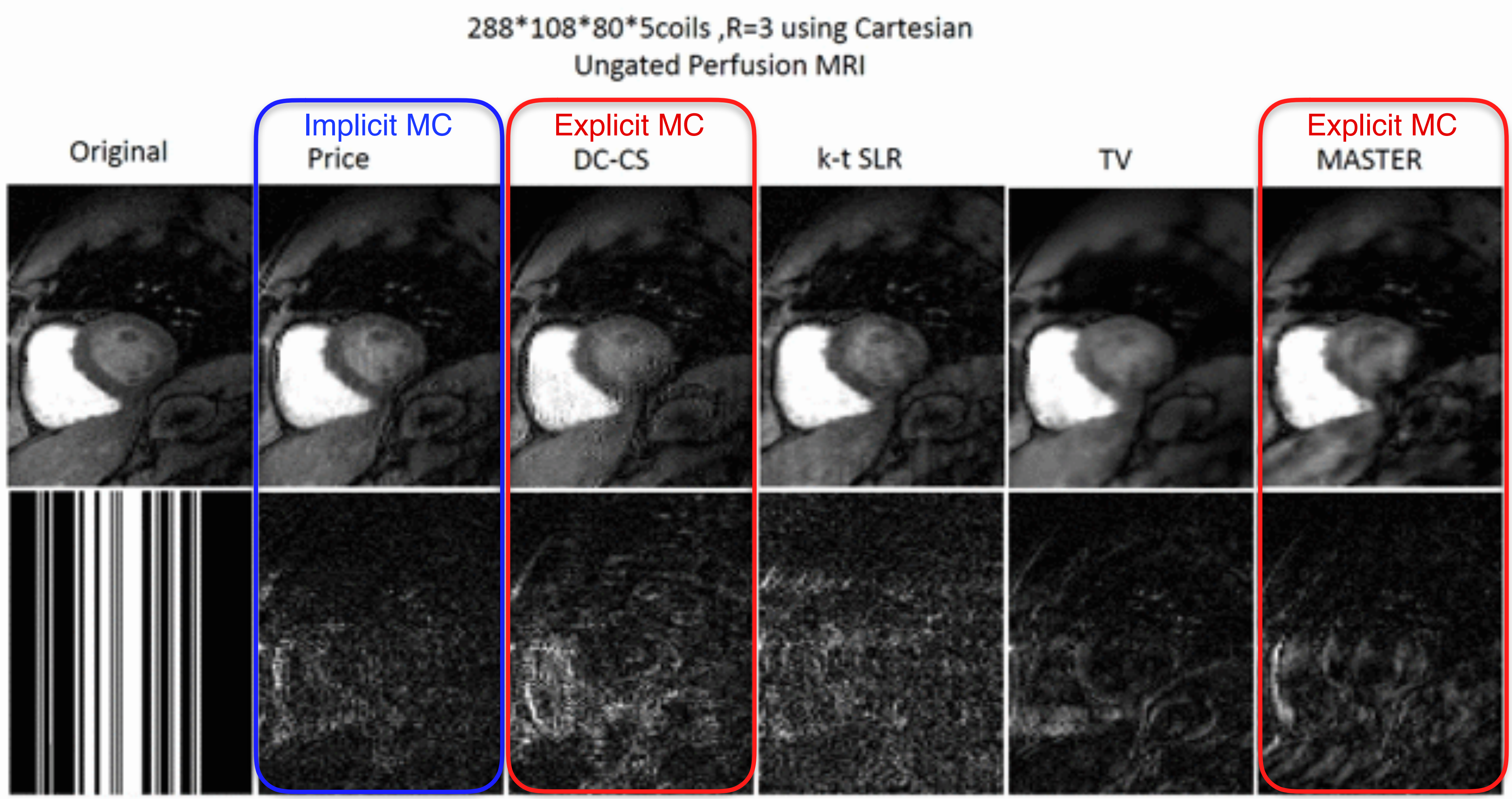

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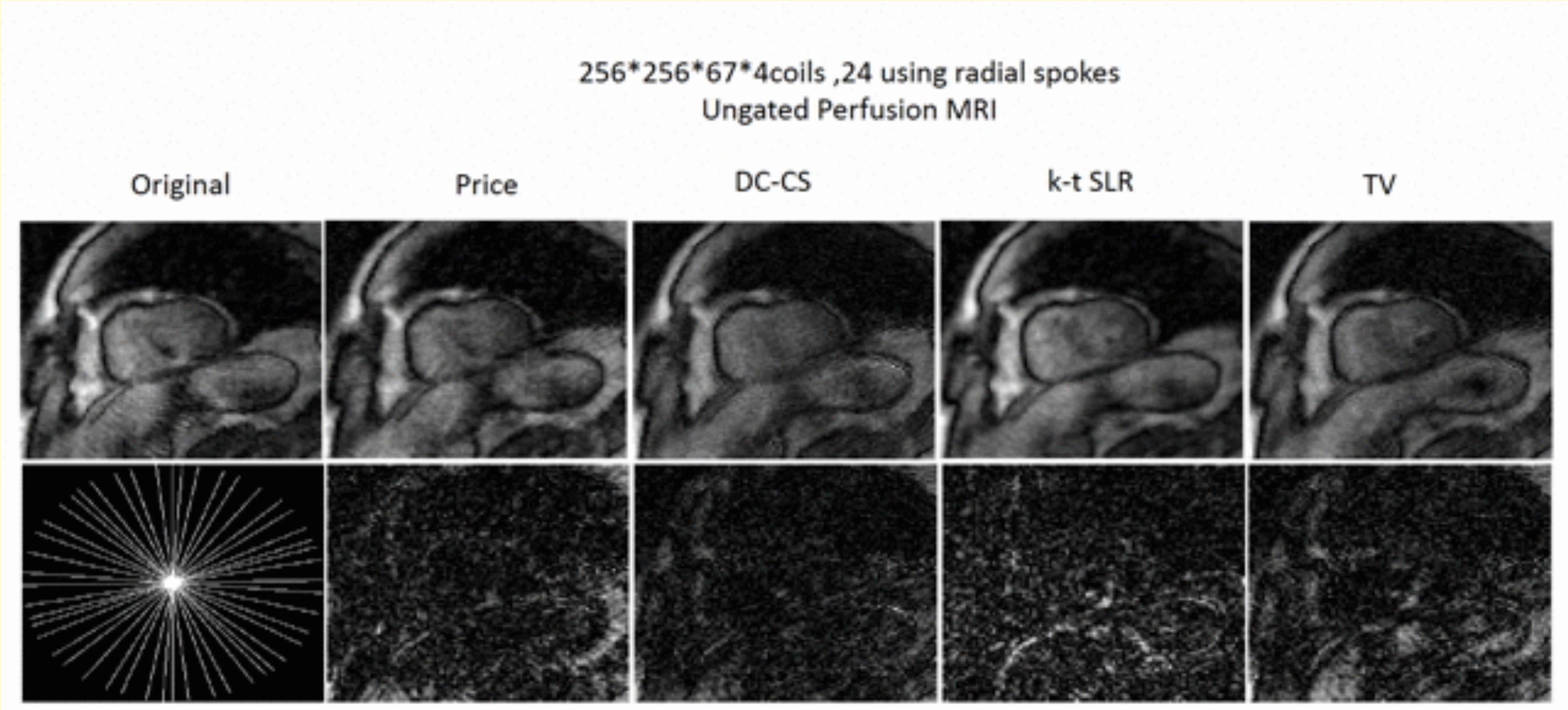

Mohsin, Lingala, Dibella & Jacob., MRM 16

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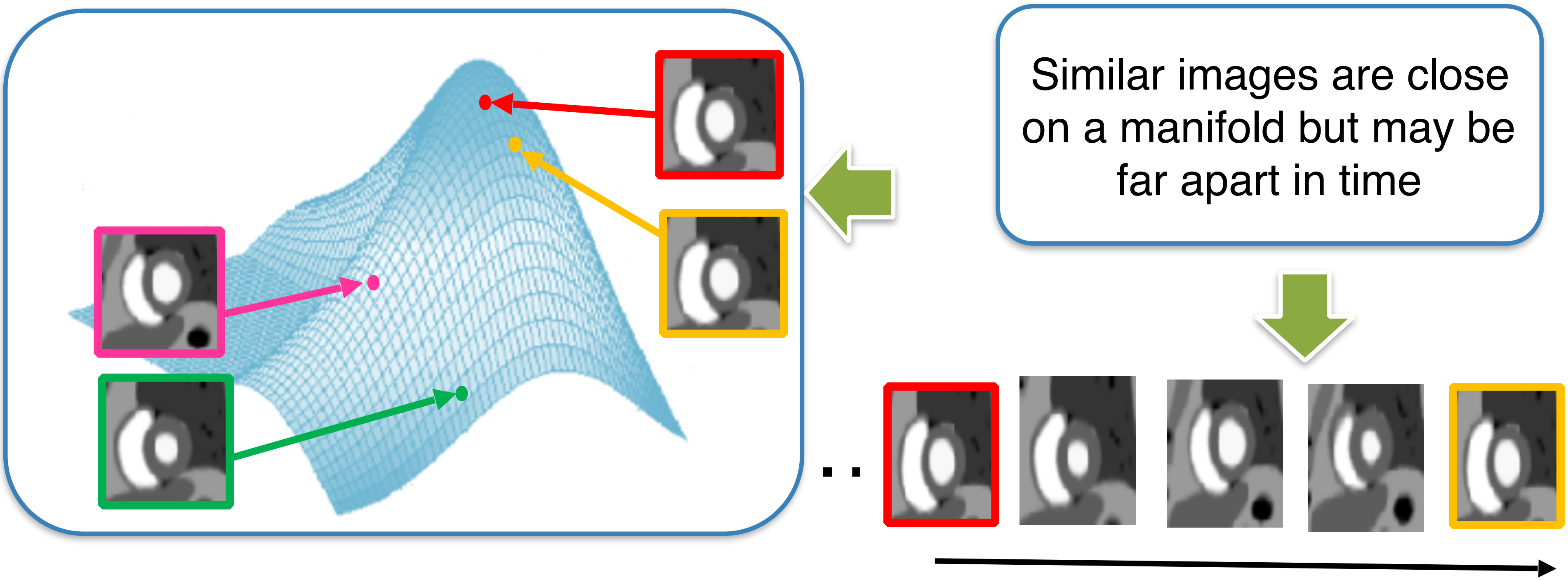
Free breathing & ungated CINE



Images: function of cardiac & respiratory phase

Manifold recovery: implicit motion resolved recon.

Manifold structure from navigators

Single step reconstruction

Estimation of manifold structure from navigators

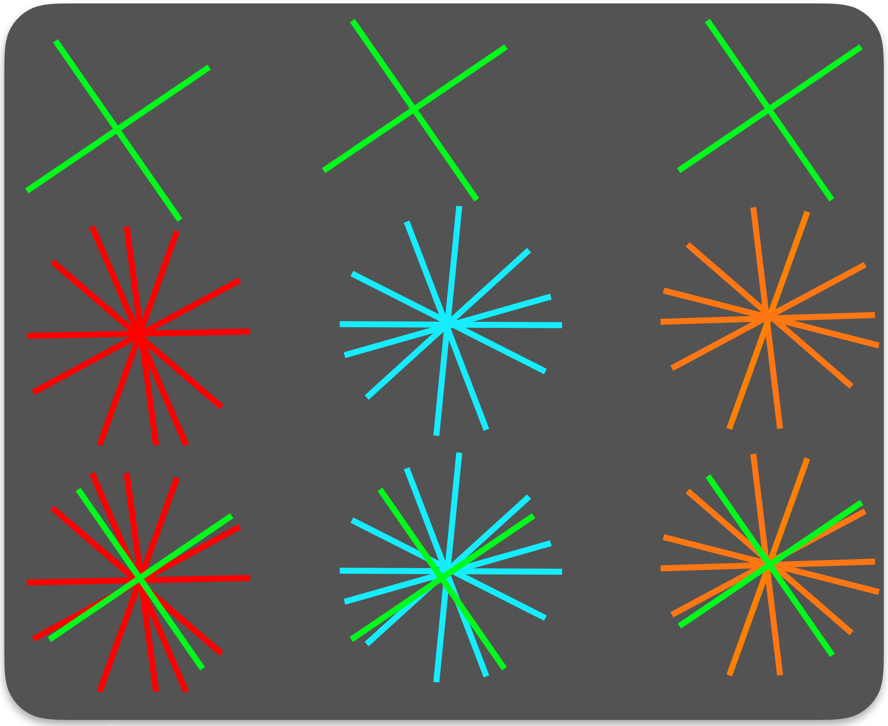
$$w_{ij} = \begin{cases} \mathbf{e}^{-\frac{\|\mathbf{z}_i - \mathbf{z}_j\|^2}{\sigma'^2}} & \text{if} \quad \|\mathbf{z}_i - \mathbf{z}_j\| < \epsilon' \\ 0 & \text{else} \end{cases}$$

Manifold smoothing of the image

$$\{\mathbf{X}^*\} = \arg\min_{\mathbf{X}} \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F^2 + \lambda \sum_i \sum_j \left(\sqrt{w_{ij}}\, \|\mathbf{x}_i - \mathbf{x}_j\|_p\right)^p$$

Navigators: each frame is collected by same pattern

$$\mathbf{y}_i = \begin{bmatrix} \mathbf{z}_i \\ \mathbf{q}_i \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} \\ \mathbf{B}_i \end{bmatrix} \mathbf{x}_i$$



$$w_{ij} = \begin{cases} e^{-\frac{\|\mathbf{z}_i - \mathbf{z}_j\|^2}{\sigma'^2}} & \text{if} \quad \|\mathbf{z}_i - \mathbf{z}_j\| < \epsilon' \\ 0 & \text{else} \end{cases}$$

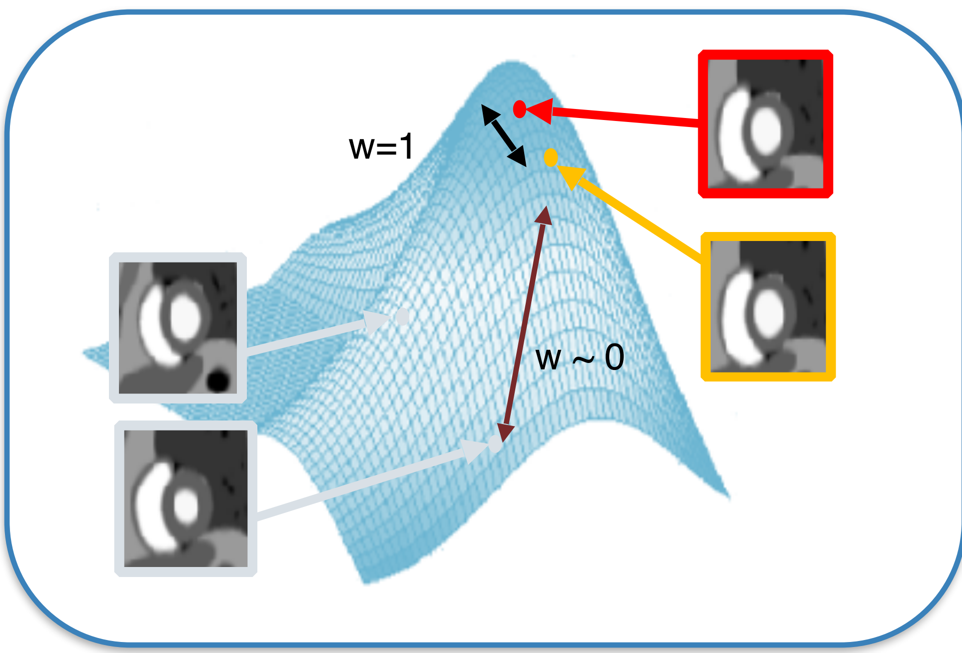
Manifold smoothness regularization

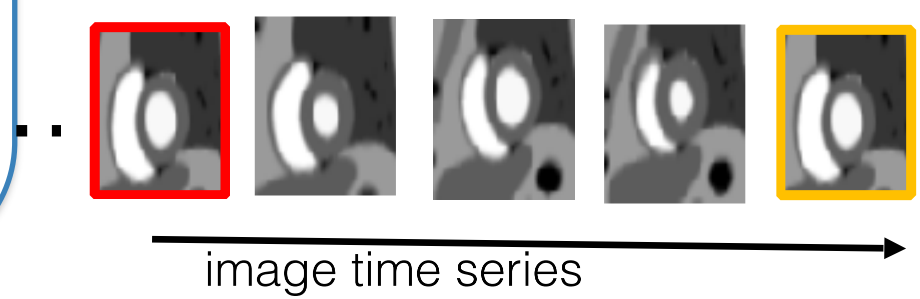
$$\{\mathbf{X}^*\} = \arg\min_{\mathbf{X}} \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F^2 + \lambda \sum_i \sum_j \left(\sqrt{w_{ij}}\; \|\mathbf{x}_i - \mathbf{x}_j\|_p\right)^p$$

Data consistency

Manifold smoothness

Solved using conjugate gradients algorithm





Manifold embedding theory

Random ortho-projection of manifold vectors

$$\mathbf{z}_i = \mathbf{\Phi}\mathbf{x}_i$$

Preserves distances with high probability

$$(1-\epsilon)\|\mathbf{x}_i - \mathbf{x}_j\|_2 \leq \|\mathbf{\Phi}\mathbf{x}_i - \mathbf{\Phi}\mathbf{x}_j\|_2 \leq (1+\epsilon)\|\mathbf{x}_i - \mathbf{x}_j\|_2$$

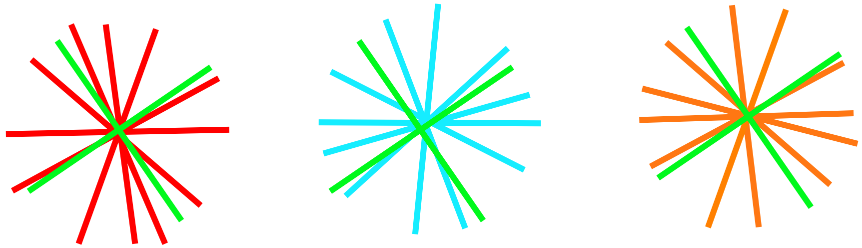
Similar to RIP property in CS

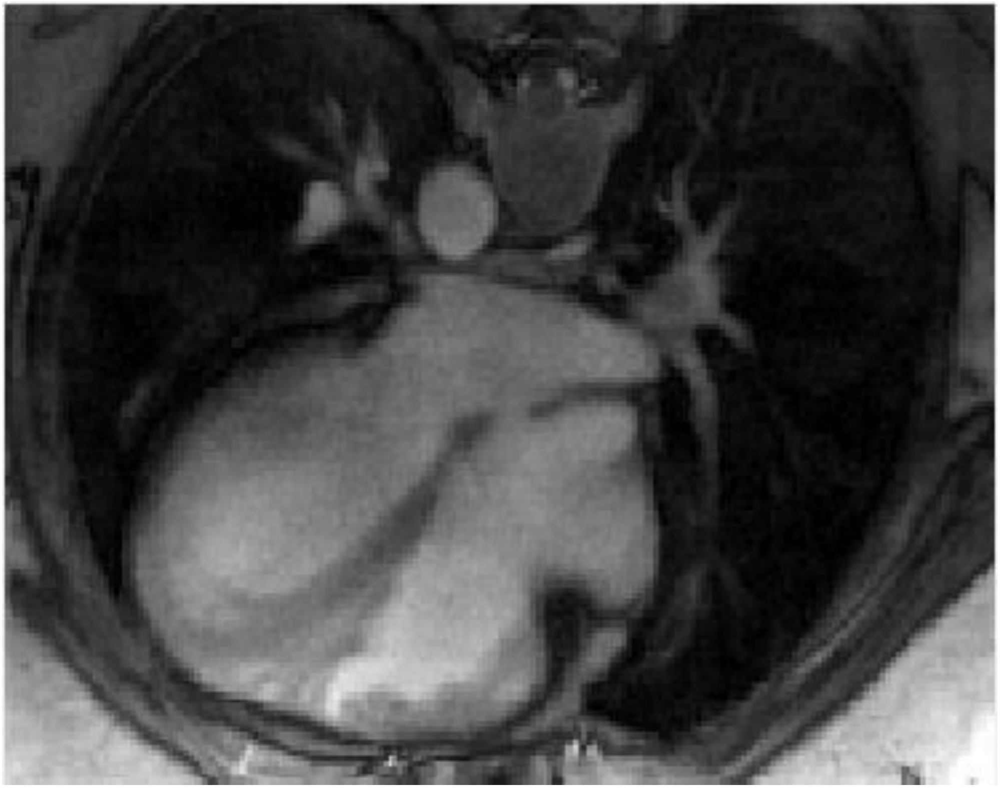
Wakin et al, 2007

Implicit motion resolved recon.

The University of Iowa

FB & UG CINE: FLASH acquisition with navigators





Aera scanner

TR=4.6ms

Temporal res=50ms

Implicit motion resolved recon.

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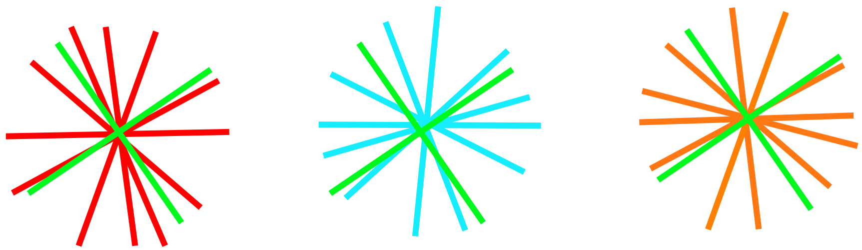
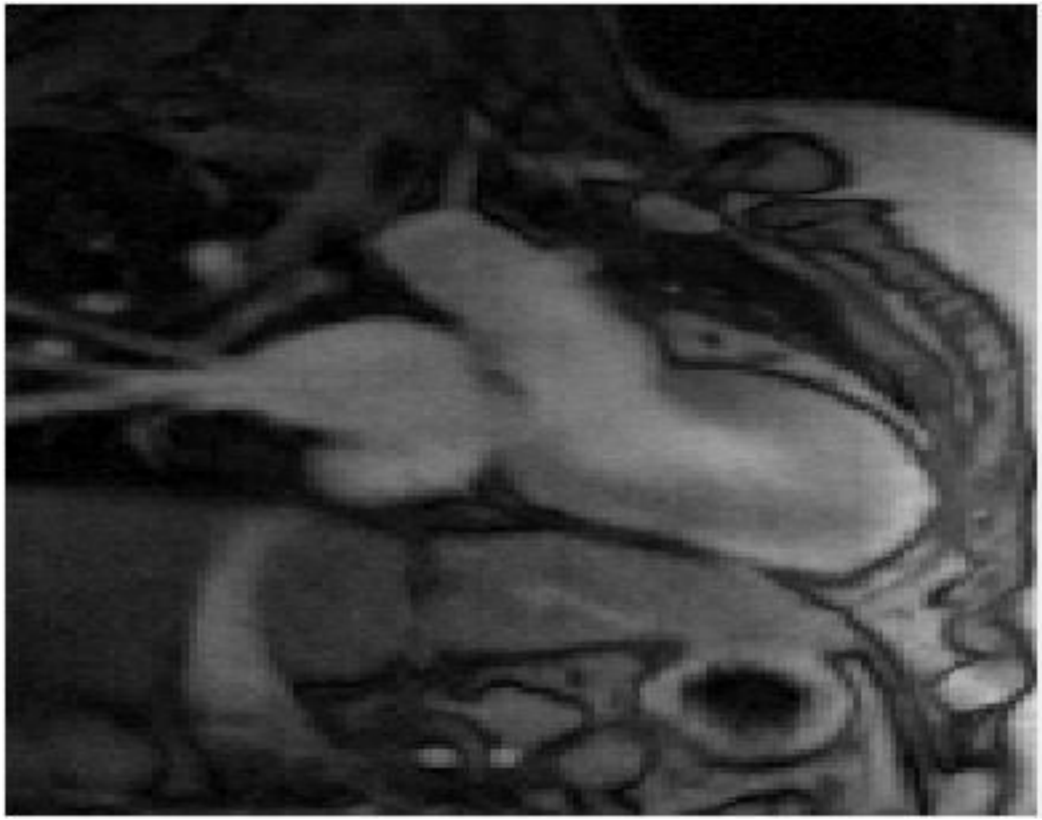
FB & UG CINE: FLASH acquisition with navigators



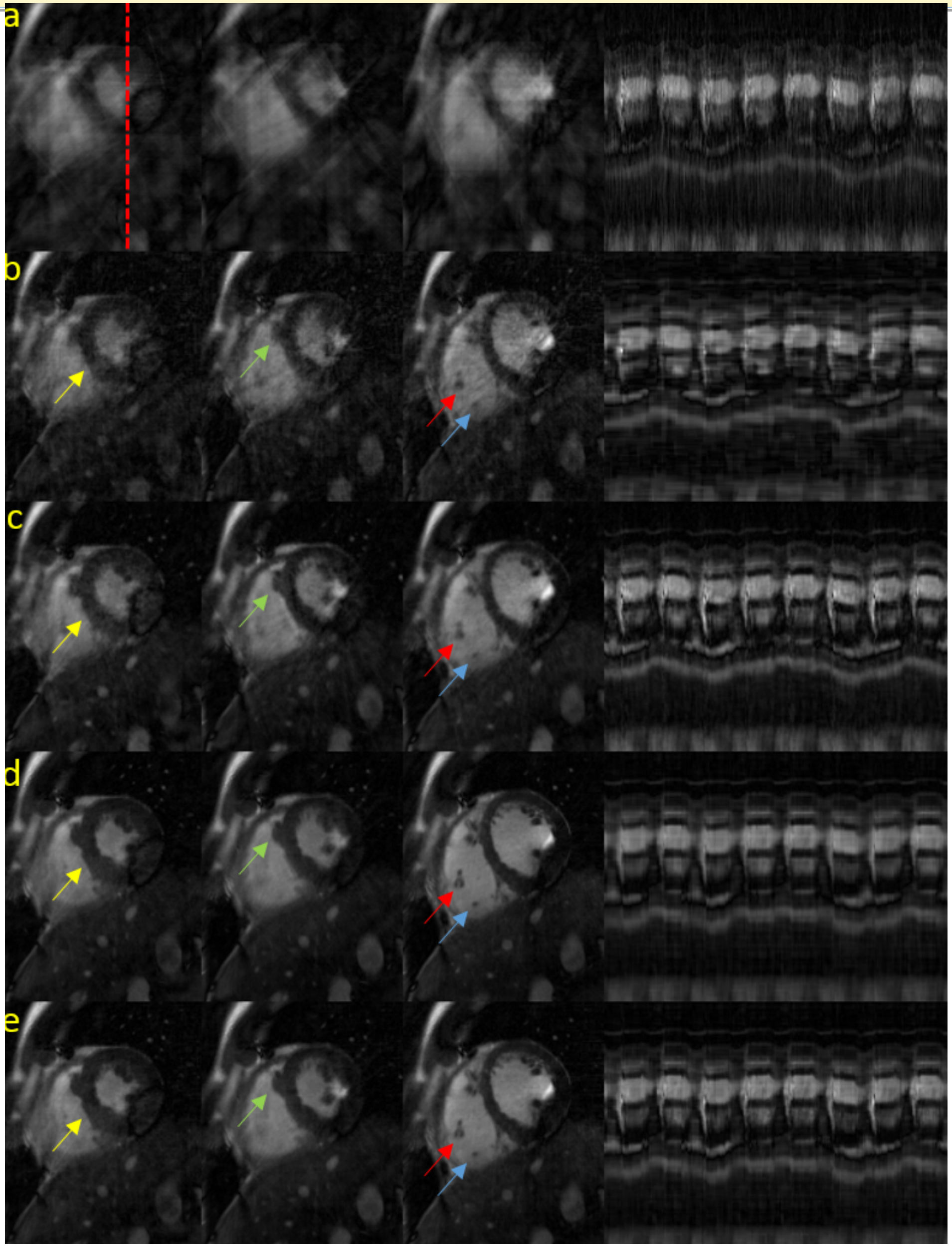


Aera scanner

TR=4.6ms

Temporal res=50ms

Comparison with other methods



View sharing

Total variation

PSF recovery

l2 manifold smoothness

l1 manifold smoothness

Tim Trio 3T scanner

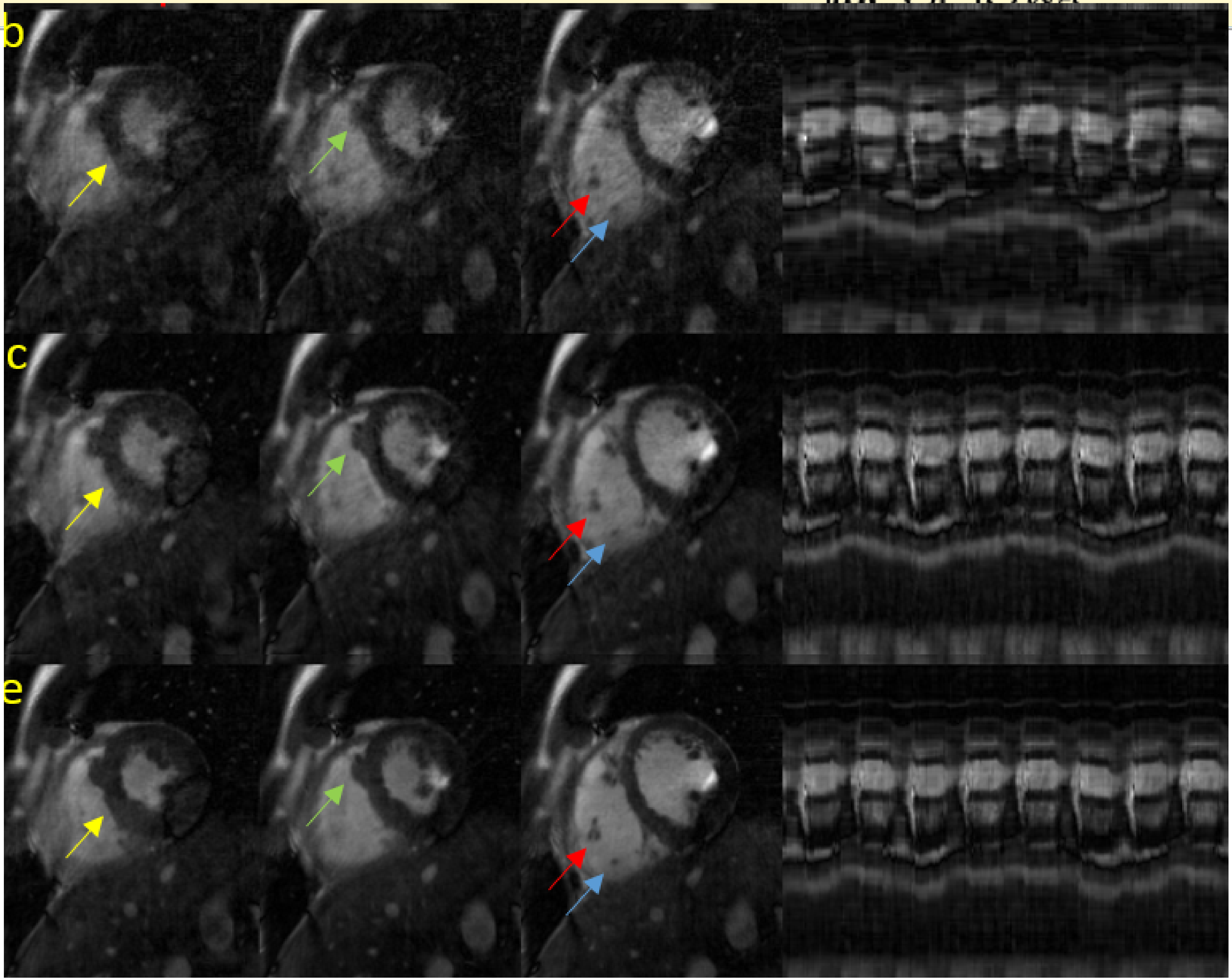
TR=4.6ms

Comparison with other methods

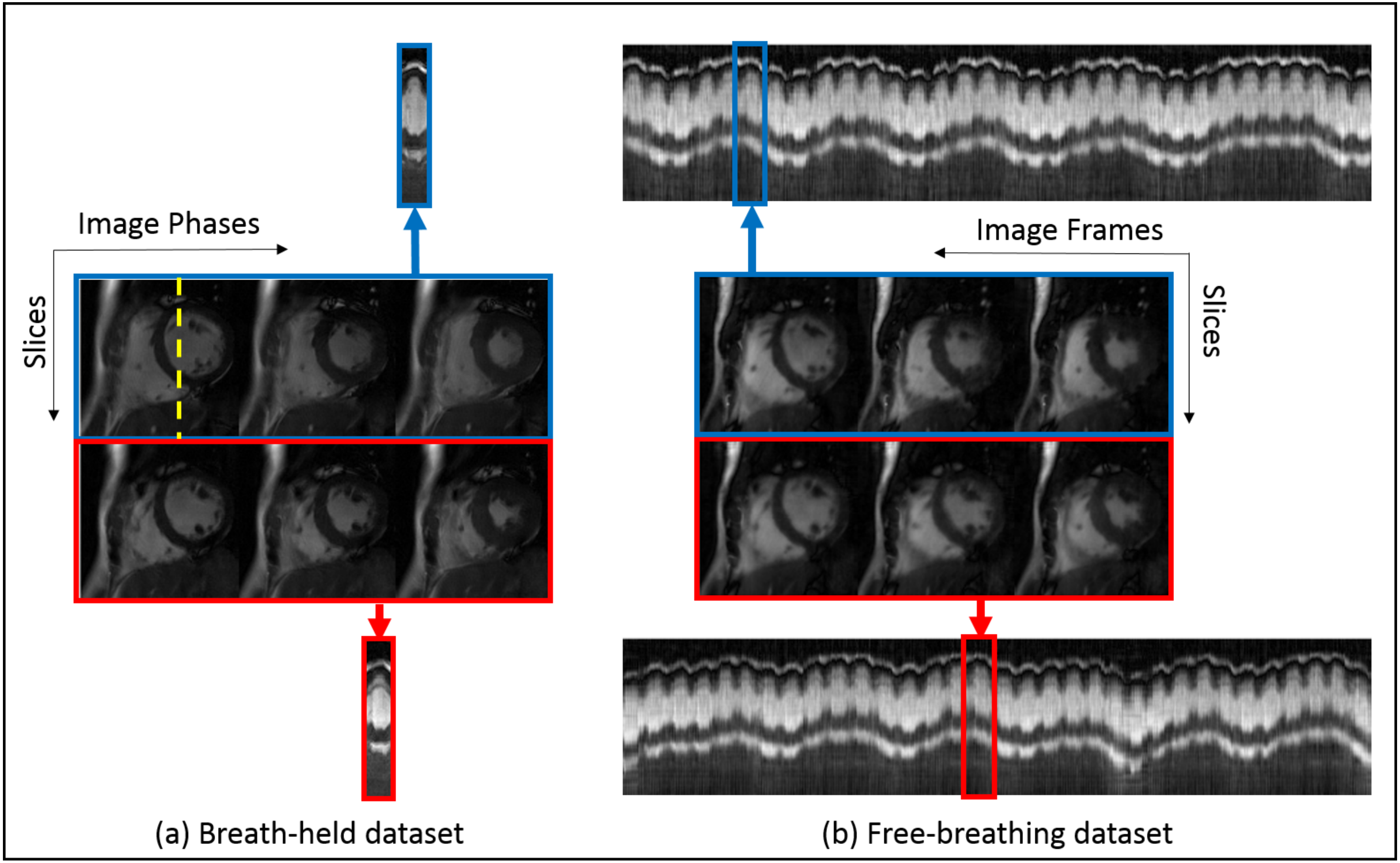
Total variation

PSF recovery

Manifold Reg



Comparison with BH acquisition



Poddar and Jacob, TMI 16

Dynamic contrast changes

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Dynamically varying contrast

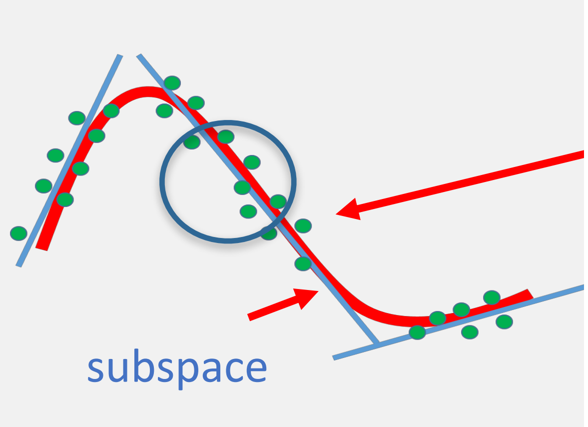
May not have sufficient neighbors

Estimate gradient matrix using sparse optimization

Graph weights: based on image similarity

$$w_{ij} = \begin{cases} \mathbf{e}^{-\frac{\|\mathbf{z}_i - \mathbf{z}_j\|^2}{\sigma'^2}} & \text{if} \quad \|\mathbf{z}_i - \mathbf{z}_j\| < \epsilon' \\ 0 & \text{else} \end{cases}$$

Not suited for flat manifold regions



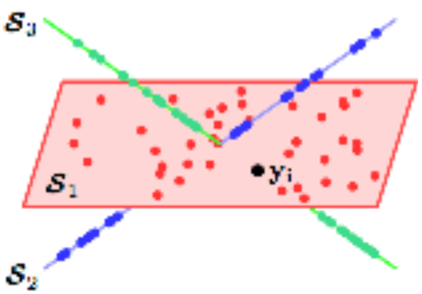
Gradient using sparse optimization

Estimate gradient matrix using sparse optimization

$$\mathbf{Q}^* = \arg\min_{\mathbf{Q}} \|\mathbf{Z}\mathbf{Q}\|^2 + \lambda\|\mathbf{Q}\|_{\ell_1} \quad \text{diag}(\mathbf{Q}) = 1; \mathbf{Q}\,\mathbf{1} = 0$$

$$\mathbf{y}_i = \begin{bmatrix} \mathbf{z}_i \\ \mathbf{q}_i \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} \\ \mathbf{B}_i \end{bmatrix} \mathbf{x}_i$$

Sparse subspace clustering [Elhamifar& Vidal]



Sparsity of Q

Represents each vector by vectors in same subspace

Myocardial perfusion MRI data

The University of Iowa

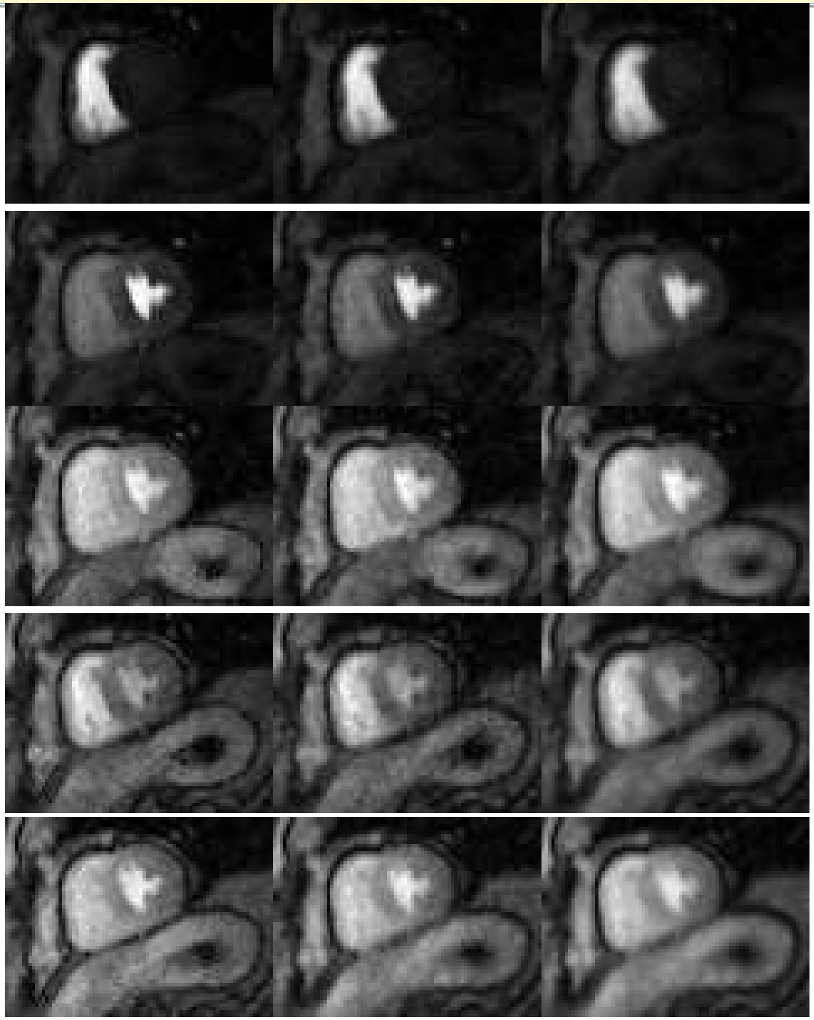
TV regularization on manifold

$$\{\mathbf{X}^*\} = \arg\min_{\mathbf{X}} \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F^2 \; + \; 2\,\lambda\, \|\mathbf{XQ}\|_{\ell_1}$$

72 lines/frame | Manifold: 24 lines/frame | PSF: 24 lines/frame

Balachandrasekharan & Jacob, ISBI 15

Myocardial perfusion MRI data

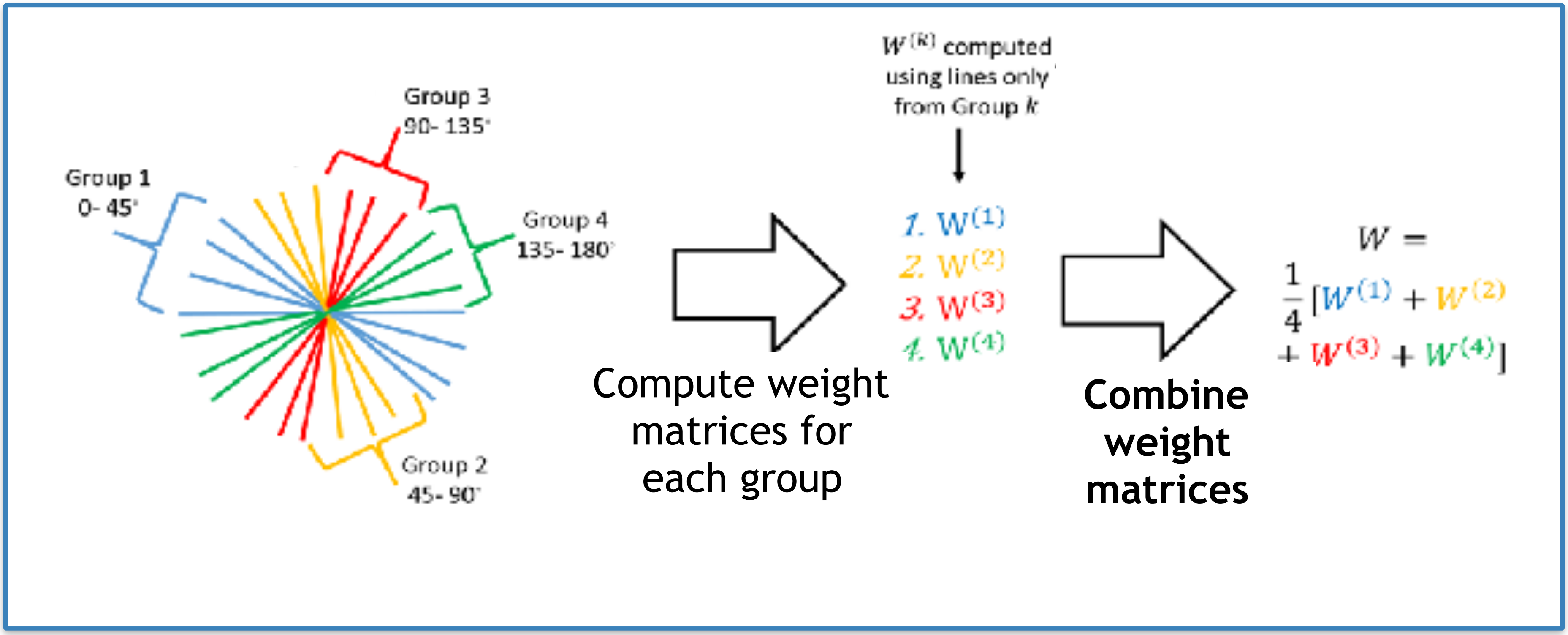


72 lines/ frame

Manifold: 24 lines/ frame

PSF: 24 lines/ frame

Towards self-gated acquisition



Poddar et al, e-Poster No 3865,Tuesday,: 08:15 - 10:15 AM

Towards self-gated acquisition

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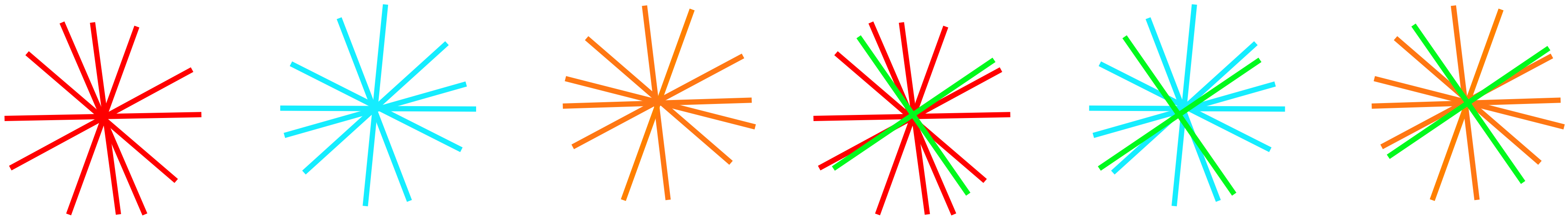
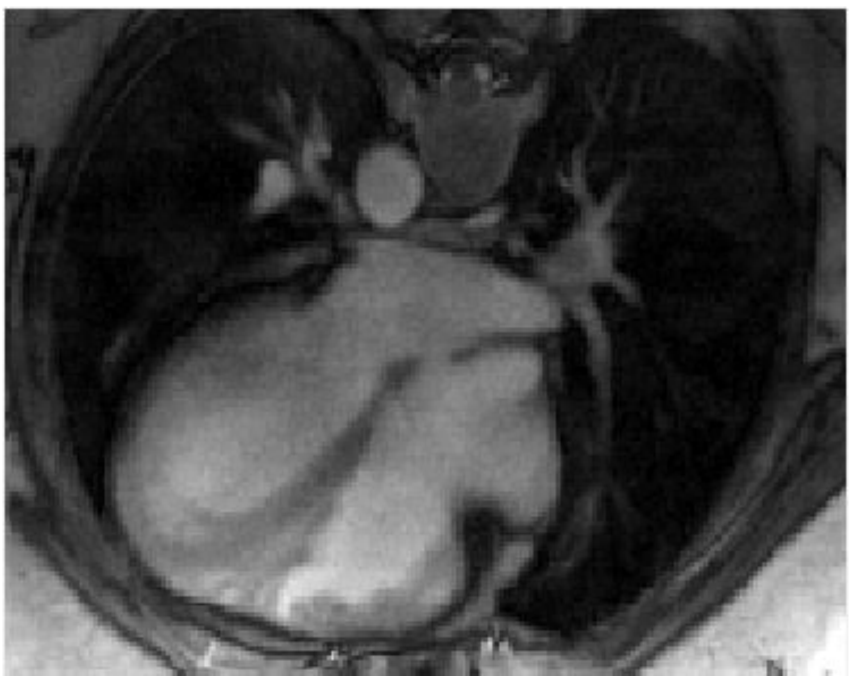
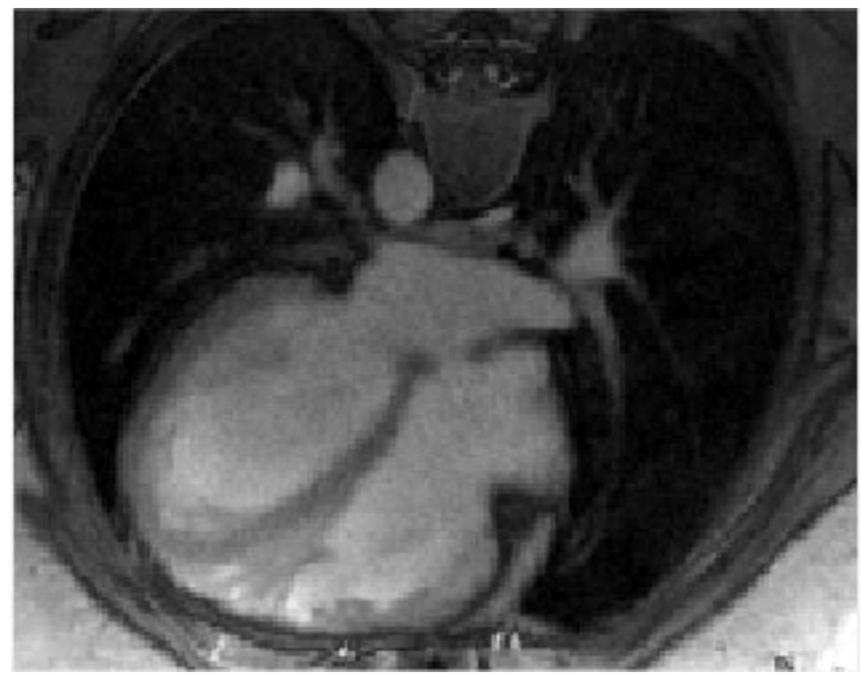




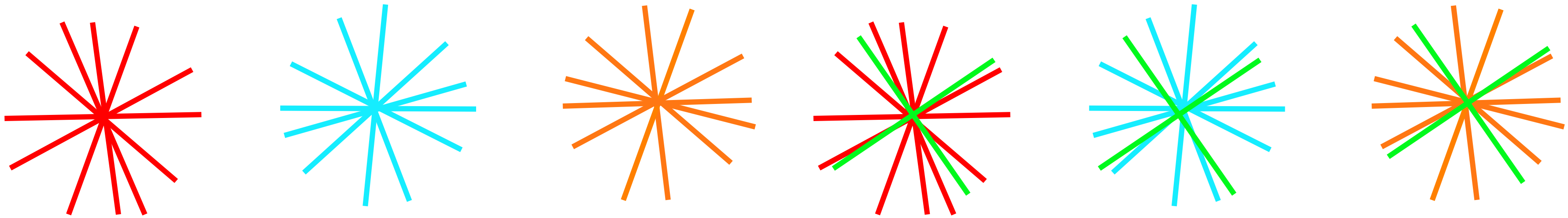
With navigators

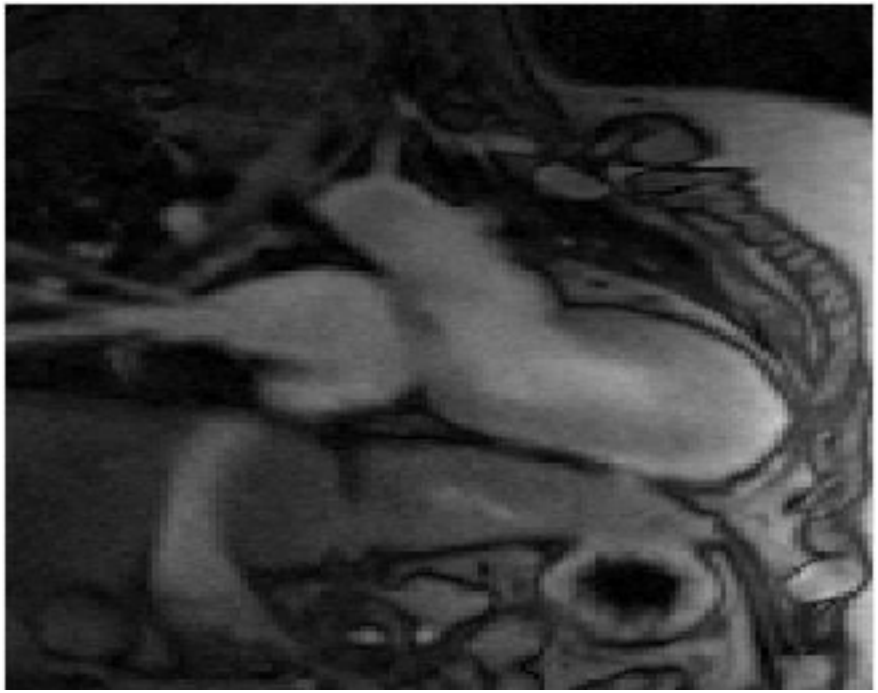


Without navigators

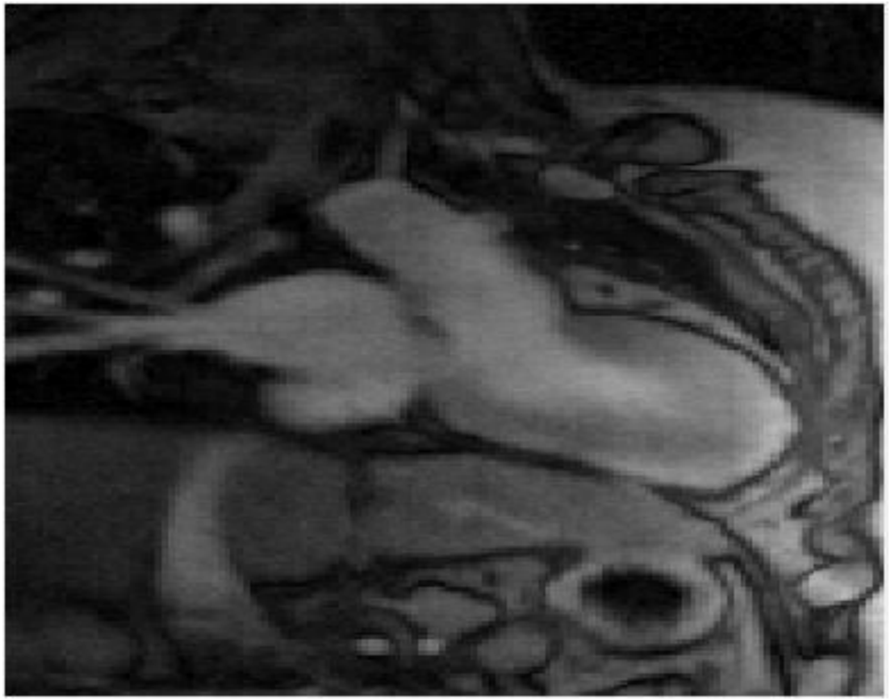
Poddar et al, e-Poster No 3865,Tuesday,: 08:15 - 10:15 AM

Towards self-gated acquisition





With navigators



Without navigators

Poddar et al, e-Poster No 3865,Tuesday,: 08:15 - 10:15 AM

Smooth manifold models: outlook

Patch manifold: implicit motion compensation

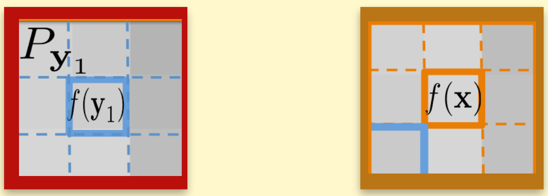
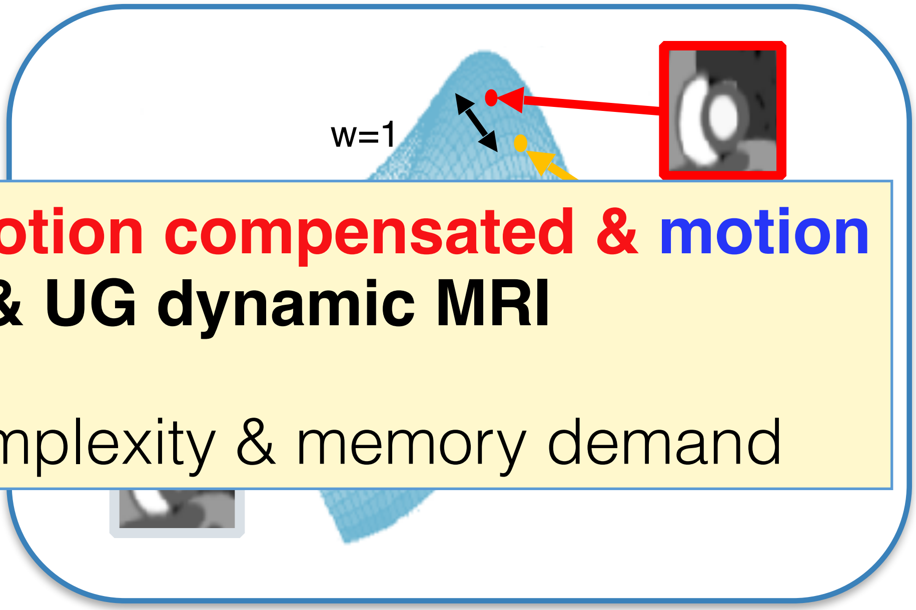
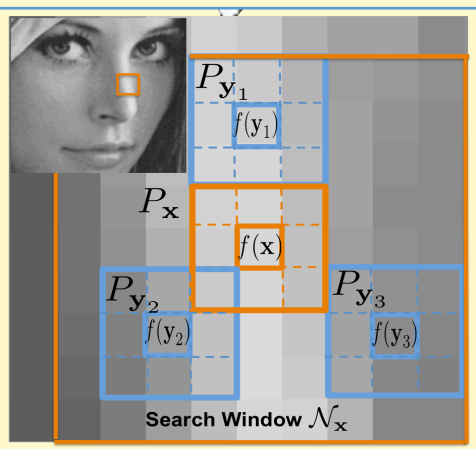


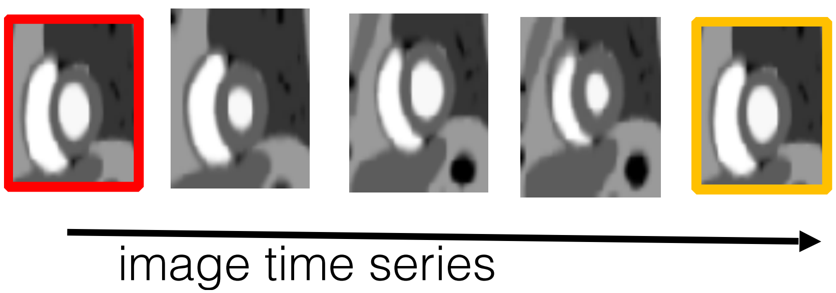
Image manifold: implicit motion resolved reconstruction



Combine the two: implicit motion compensated & motion resolved 4-D FB & UG dynamic MRI

Challenges: computational complexity & memory demand





Kernel PCA

Kernel PCA: PCA on non-linear features

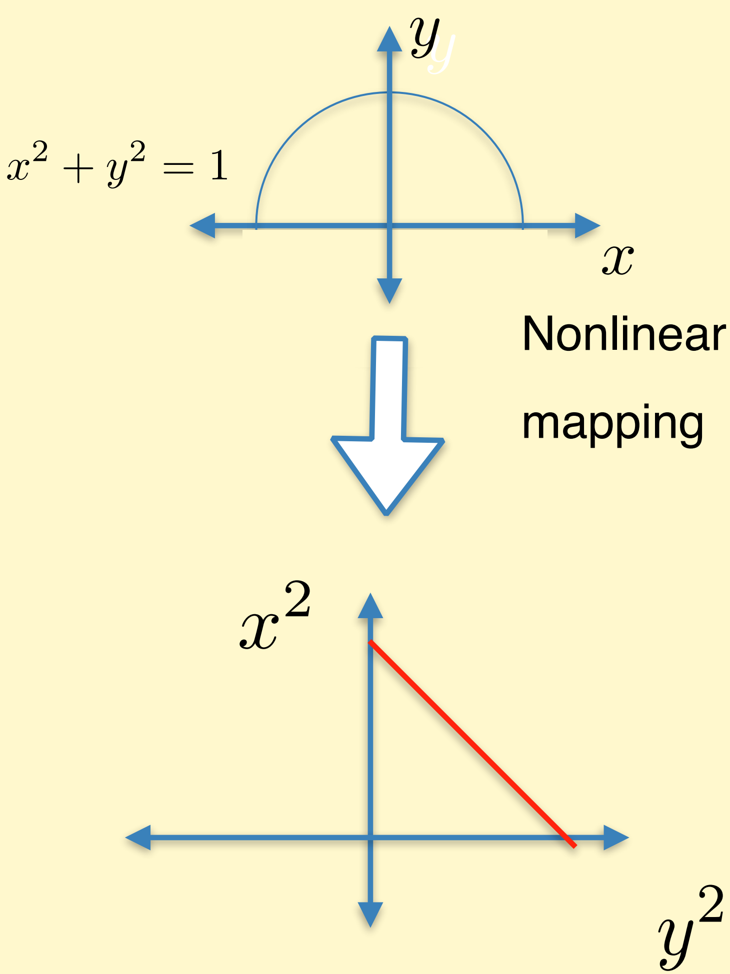
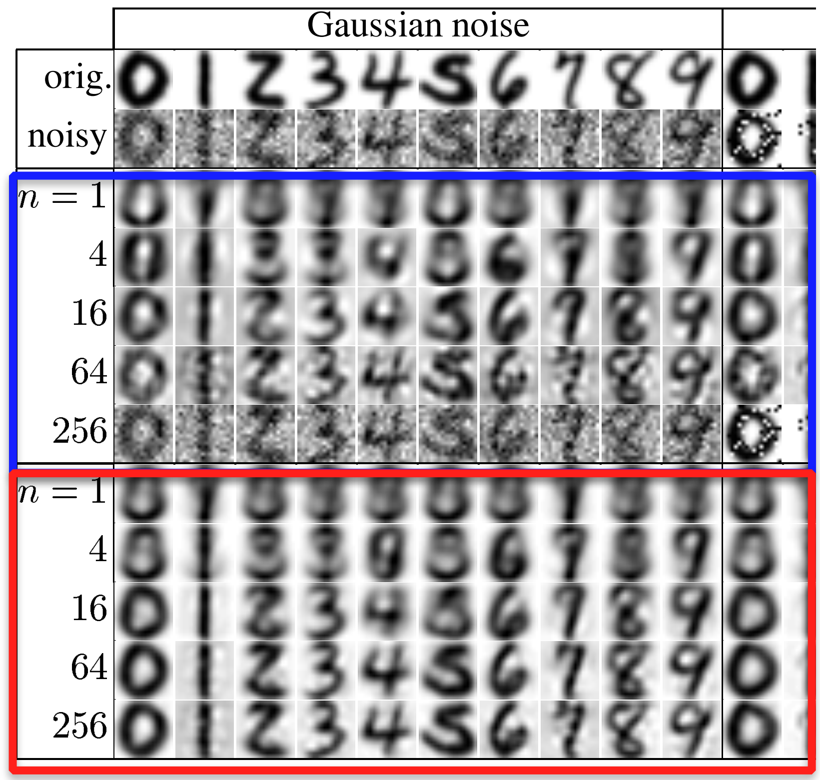


Image denoising

Linear PCA

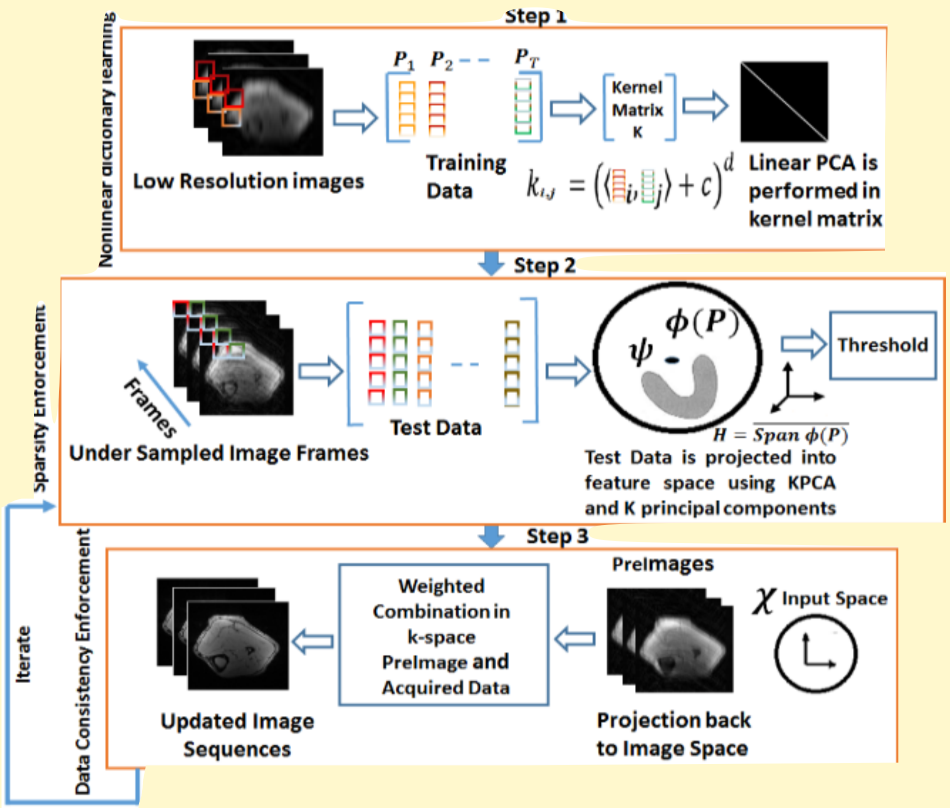


K-PCA

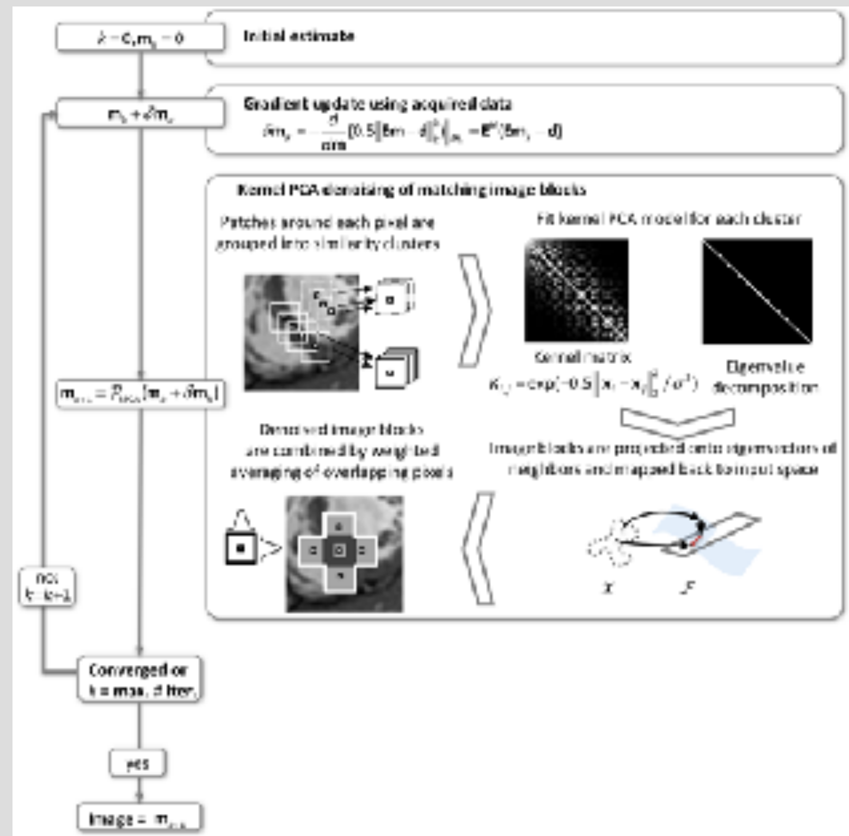
Mika et al, NIPS 99

Application to MRI

Few basis functions in non-linear space



Nakarmi & Ying, 2015



Schmidt et al, 2016

Relation to Kernel PCA

Manifold smoothness regularization

$$\{\mathbf{X}^*\} = \arg\min_{\mathbf{X}} \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F^2 + \lambda \sum_i \sum_j \left(\sqrt{w_{ij}}\, \|\mathbf{x}_i - \mathbf{x}_j\|_p\right)^p$$

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F^2 + 2\lambda\ \mathrm{trace}(\mathbf{X}\mathbf{L}\mathbf{X}^H),$$

Graph Laplacian

$$\mathbf{L} = \mathbf{D} - \mathbf{W}$$

Diagonal matrix

$$\mathbf{D}(i,i) = \sum_j \mathbf{w}_{ij}$$

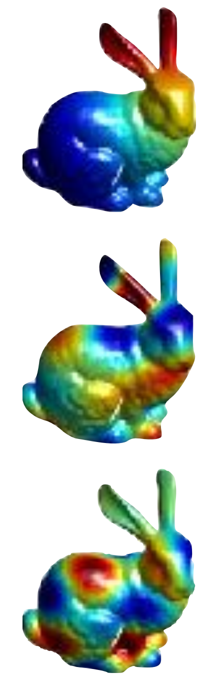
Kernel matrix

Manifold smoothing & KPCA

Eigen decomposition: Fourier transform on graphs

$$\mathbf{L} = \mathbf{V}\mathbf{\Sigma}\mathbf{V}^H$$

V: orthogonal basis (graph Fourier exponentials)



Relation to k-t PCA/PSF methods

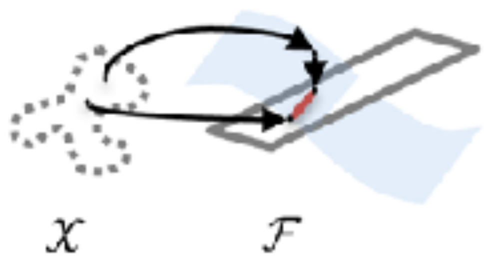
$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F^2 + 2\lambda\ \mathrm{trace}(\mathbf{X}\mathbf{L}\mathbf{X}^H),$$

⇩

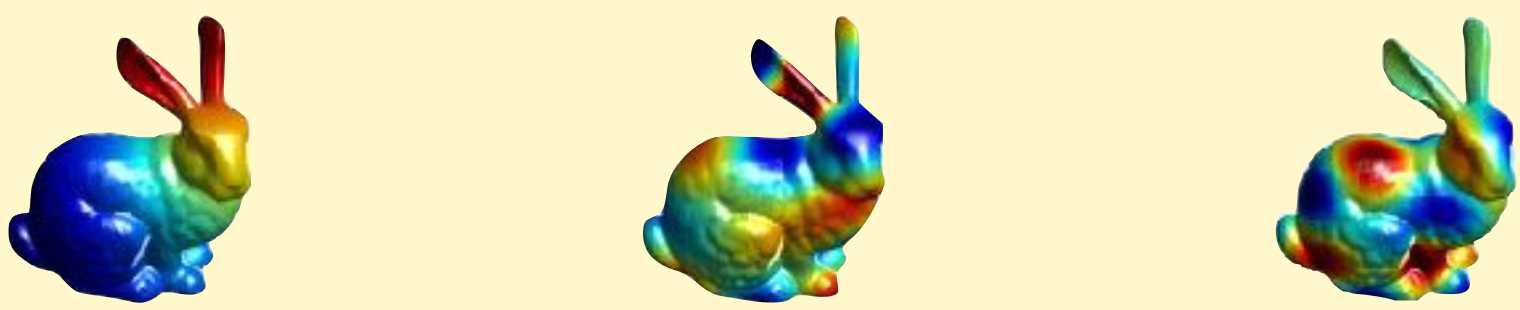
$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F^2 + 2\lambda \sum_i \sigma_i \|\mathbf{U}_i^H \mathbf{X}\|_F^2$$

Relation to Kernel PCA

KPCA: Minimum energy representation on manifold



Smoothness regularization: smooth manifold



Summary

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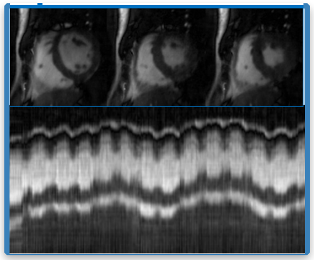

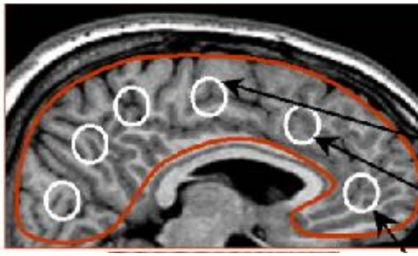

Signals on smooth manifold

High rank matrix

Patch manifold: robust distance minimization

First iteration similar to non-local means

Dynamic MRI: implicit motion compensation

Image manifold: robust distance minimization

Dynamic MRI: implicit motion resolved reconstruction

Software available

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https://research.engineering.uiowa.edu/cbig

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Software

MATLAB software

1. k-t SLR: Accelerated dynamic MRI using low rank and sparse penalties
2. HDTV: Higher degree total variation regularization
3. Generalized HDTV : Fast implementation of HDTV regularization for 3D inverse problems
4. Optimized NUFFT: Non-uniform fast Fourier transfor for nonCartesian MRI
5. BCS/Blind CS: Blind compressed sensing dynamic MRI
6. GOOSE: GlObally Optimal Surface Estimation for fat water decomposition
7. (DC-CS): Deformation corrected compressed sensing dynamic MRI
8. PatchReg: Iterative Shrinkage Algorithm for Patch-Smoothness MRI
9. PRICE: Patch Regularization for Implicit motion CompEnsation