Novel annihilation filter framework for accelerated parameter mapping

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Synopsis

Quantitative parameter maps offer valuable information about various tissue attributes, which are early markers for many neurological disorders. However the long acquisition time of the associated image time series puts a restriction on the achievable spatial resolution. In this work, we introduce a novel framework, which exploits the exponential nature of the time profiles at every pixel and spatial smoothness of the exponential parameters to recover the images from highly under-sampled measurements. Our preliminary results clearly demonstrate the potential of the proposed algorithm.

Purpose

The estimation of exponential parameters (e.g frequency, relaxation parameters) from time series data is a key problem in several MRI applications including parameter mapping, spectroscopic imaging, field mapping, and fat water imaging. The acquisition of multiple images at different settings (e.g. TE, TR, spin-lock duration) is associated with increased scan time, which often restricts the achievable spatial resolution. A common approach to overcome these problems is to acquire under sampled data and regularize the reconstruction using appropriate priors (e.g sparsity low rank etc.).[1, 2] However, most of the current regularization priors do not exploit the exponential behavior of the time-series and the spatial smoothness of the parameters. The main focus of this work is to introduce a novel annihilation filter formulation, which directly exploits the exponential structure of the time series and the spatial smoothness of the exponential parameters.

Methods

Our work is centered on the linear predictability of a 1-D exponential time-series. This implies that a linear combination of damped/undamped exponentials can be linearly predicted/annihilated by the convolution with a 1-D filter, whose degree is equal to the number of exponentials at that pixel[3]. We model the temporal signal at each voxel location $\mathbf{r}$ as

$$\rho[\mathbf{r}, n] = \sum_{i=1}^{L} \alpha_i(\mathbf{r})\, \beta_i(\mathbf{r})^n.$$

In T_2 mapping applications, the exponential parameters $\beta_i = \exp\left(\frac{-\Delta T}{T_{2,i}(\mathbf{r})}\right)$, where ΔT is the time between two image frames and $T_{2,i}$ is the relaxation parameter of the i^{th} tissue component. Such a signal can be linearly predicted/annihilated by the convolution with a 1-D filter $h[\mathbf{r}, z] = \prod_{i=1}^{L}(1 - \beta_i(\mathbf{r}) z^{-1})$. The exponential parameters often vary smoothly in space; we exploit this fact by constraining the filter coefficients to be spatially band-limited. This results in the following 3-D convolution relation:

$$\hat{\rho}[\mathbf{k}, n] \otimes d[\mathbf{k}, n] = 0,$$

where $\otimes$ is the 3-D convolution operator, $\rho[\mathbf{k}, n]$ and $d[\mathbf{k}, n]$ are the k-t space coefficients of the image time-series $\rho[\mathbf{r}, n]$ and the annihilation filter $h[\mathbf{r}, n]$ respectively. The extent of the filter $d[\mathbf{k}, n]$ along the spatial frequency and temporal dimension controls the spatial smoothness of the filters and the exponential parameters respectively. The above annihilation relations can be compactly represented in a matrix form as $\mathcal{T}(\hat{\rho})\,\mathbf{d} = 0$, where $\mathcal{T}(\hat{\rho})$ is a block Toeplitz matrix and $\mathbf{d}$ is the vectorized filter coefficients. See figure (1) for the construction of the Toeplitz matrix. The annihilation relation implies that the matrix $\mathcal{T}(\hat{\rho})$ is low-rank. Hence, we formulate the recovery of ρ from undersampled Fourier measurements as the following structured low rank matrix recovery problem in the Fourier domain:

$$\hat{\rho}^{\star} = \arg\min_{\hat{\rho}} \|\mathcal{T}(\hat{\rho})\|_p + \frac{\lambda}{2} \|\mathcal{A}(\hat{\rho}) - \mathbf{b}\|_2^2$$

where $\hat{\rho}$ are the 2-D Fourier coefficients of the volume ρ, $\mathbf{b}$ represents the undersampled measurements, $\mathcal{A}$ is a linear operator that encodes the coil sensitivity and the Fourier undersampling matrices and λ is a regularization parameter. To solve the above equation, we employ an Iterative Reweighted Least Squares (IRLS) based algorithm[4].

We demonstrate the algorithm on the recovery of both single channel (coil compressed) and multi-channel data from under-sampled Fourier measurements. For this purpose, a fully sampled 2-D dataset was acquired using a turbo spin echo sequence and the following scan parameters were used: Matrix size - 128x128, Coils = 12, FOV: 22x22 cm^2, TR = 2500 ms and slice thickness = 5mm. The T_2 weighted images were obtained for 12 equispaced echo times ranging from 10 to 120ms. Post image recovery, the T_2 maps were estimated by fitting a mono-exponential model to each voxel.

Results

Figure (2) compares the proposed approach with the k-t Low Rank algorithm[2] (k-t SLR without the sparsity) on the recovery of single channel data from 30$\%$ uniform random measurements. In figure (3), we compare the two methods on the recovery of multi-channel data from 12-fold (3-fold variable density + 4-fold 22 Cartesian) undersampled measurements. In both cases, We observe that the reconstructions from the proposed scheme have lower errors (see caption for details). In figure (4), we study the impact of the filter sizes on the quality of the reconstructions. We observe that the errors in the reconstructions are reduced when the spatial support of the filter is decreased, which clearly demonstrates the benefit of exploiting spatial smoothness of the exponential parameters.

Conclusion

We introduced a novel annihilation filter framework to recover the T_2 weighted MR images from under-sampled Fourier measurements. The reconstructions and the T_2 maps from the proposed approach have fewer errors compared to the low rank method. Also, the benefit of exploiting the spatial smoothness of the exponential parameters was demonstrated through the use of smaller filters, which helped improve the reconstruction quality.

Acknowledgements

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References

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4. Mohan, Karthik, and Maryam Fazel. "Iterative reweighted least squares for matrix rank minimization." Communication, Control, and Computing (Allerton), 2010 48th Annual Allerton Conference on. IEEE, 2010.

Figures

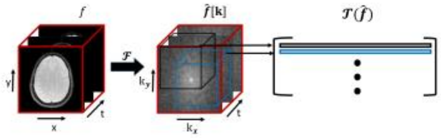

figure (1): Construction of the Toeplitz matrix $\mathcal{T}(\hat{\rho})$: The rows of the matrix correspond to the cube shaped neighborhoods of the Fourier samples. The size of the cube is equal to the size of the filter. The missing entries of the matrix are estimated by exploiting its low rank structure. The smoothness of the exponential maps and the number of exponentials can be controlled by the size of the cubes.

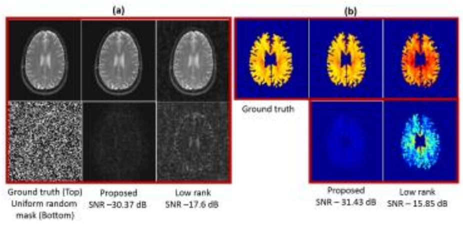

figure (2): Comparison of the proposed method with k-t Low rank on the recovery of single channel data from 30 percent uniform random measurements. For the proposed approach, we chose a filter of size 122x122x2 and a Schatten p = 0.6 was chosen for both the methods. The improvements offered by the proposed scheme can be easily appreciated from the estimated T_2 error images in (a) and the T_2 maps in (b).

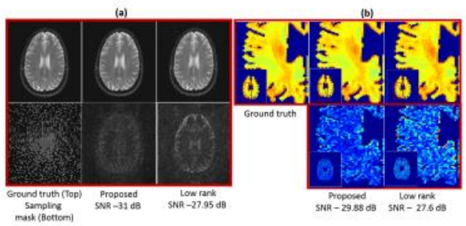

figure (3): Comparison of the proposed method with $k-t$ low rank on the recovery of multi-channel data at an acceleration of 12. For the proposed approach, we chose a filter of size 114x114x10 and a Schatten p = 0.7 was chosen for both the methods. We observe that the reconstructions from the proposed method have fewer errors, which can be appreciated from the error maps of the T_2 weighted images in (a) as well with the noise-like artifacts in the T_2 maps in (b).

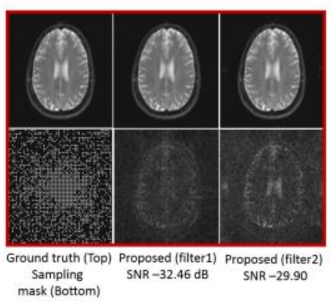

figure (4): Comparison of reconstructions from 8-fold undersampled data using 118x118x10 (filter1) and 128x128x10 (filter2) respectively: The errors in the reconstructions using filter1 are reduced clearly demonstrating the advantage of having a smaller support for the filter.