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Distributed control for optimal economic dispatch of power generators: the heterogenous case

Raghuraman Mudumbai, Soura Dasgupta, and Brian B. Cho

***Abstract*—In this paper, we present a simple, distributed algorithm for frequency control and optimal economic dispatch of power generators. In this algorithm each generator independently adjusts its power-frequency set-points of generators to correct for generation and load fluctuations using only the aggregate power imbalance in the network, which can be observed by each generator through local measurements of the frequency deviation on the grid. In the absence of power losses, we prove that the distributed algorithm eventually achieves optimality i.e. minimum cost power allocations, under mild assumptions (strict convexity and positivity of cost functions); we also present numerical results from simulations to compare its performance with traditional (centralized) dispatch algorithms. Furthermore, we show that the performance of the algorithm is robust in the sense that even with power losses it corrects for frequency deviations, and for low levels of losses, it still achieves near-optimal allocations; we present an approximate analysis to quantify the resulting suboptimality.**

I. Introduction

We present a simple distributed algorithm for load frequency control and economic dispatch in an electric grid; in this algorithm each generator uses local knowledge of its own cost of generation along with measurements of frequency deviations to dynamically adjust its real power generation. We show that over the course of network operation, the algorithm eventually achieves the condition of optimal economic dispatch under mild assumptions on the cost functions of the generators when there are no line losses (and near-optimal allocations as long as the line losses are not too large). This generalizes our earlier work in [1] where optimality was achieved only when all generators had the same underlying production cost i.e. identical cost functions.

A. Motivation

This work is motivated by the anticipated needs of the next generation electric grid which is expected to have smart consumer end-nodes [2] and a high penetration of alternative energy generators. Since the availability of alternative energy sources such as wind and solar generators is inherently intermittent in time and dispersed in geography [3], the ability of the electric grid to dynamically adjust generation and consumption is key to achieving a high load factor and efficient energy use. Decentralized control techniques are well-suited to such a flexible electric grid, and this recognition has led to an increased interest in concepts such as microgrids [4] and distributed generation (DG) [5]. Similarly, decentralized control techniques are extremely attractive to "smart grids" [6], where there is the possiblity of controlling loads in addition to generation, in response to real-time surplus or scarcity of power in the electric grid.

In a traditional electric grid, control of generators, i.e. Automatic Generation Control (AGC)[1] is accomplished on multiple time-scales using multiple different mechanisms [9]. Primary control is implemented in a distributed fashion at the generators, but secondary and tertiary control (corresponding to load frequency control (LFC) and economic dispatch (ED) respectively) are implemented from a centralized control station at the Load Serving Entity (LSE) and transmission system operator (TSO) [10]. The goal of the secondary control process is to reduce the Area Control Error (ACE) to zero. The ACE is a measure of the imbalance between rated generation capacity and power consumed within the control area, and the LFC algorithm adjusts the power generation levels in order to achieve power balance within the control area.

Traditionally, an *ad hoc* allocation is used by the secondary controller to return ACE to zero without consideration of cost minimization; the latter function is the responsibility of the tertiary control process or economic dispatch (ED). The economic dispatch process periodically re-allocates the total power among generators to minimize total cost. Once set by the dispatch algorithm, the power allocations may deviate over time from their optimal values because of cumulative load fluctuations and the actions of the secondary controller. The dispatch problem is typically formulated as a multivariable constrained optimization problem [11] that is then solved using Lagrangian techniques such as "lambda iteration" [12]. However, when line losses are included in the model, the dispatch problem becomes analytically intractable even with simplified models for the generator cost functions. In previous academic work on the dispatch problem, complex numerical optimization methods such as genetic algorithms, particle swarm optimization or Monte-Carlo methods [13], [14] are often employed to determine the minimum cost allocation of power across generators. In contrast, in the algorithm described in this paper, there is no centralized dispatcher;

Department of Electrical and Computer Engineering, University of Iowa, Iowa City, IA 52242. Email: [rmudumbai@engineering.,dasgupta@engineering.,brianbumseok-cho@]uiowa.edu. Supported in part by US NSF grants ECS-0622017, CCF-072902, and CCF-0830747.

[1]Terms such as AGC [7], [8] may be used to mean slightly different things in the technical literature from different parts of the world, partly mirroring differences in the structure of the electric grid itself between Europe, North America and so on. In this paper, AGC is used as a generic term to include the hierarchy of mechanisms for frequency regulation, tie-line power flow control and economic dispatch.

instead each generator adjusts its own generation power iteratively and independently from other generators and this procedure is shown (under some conditions) to quickly correct for frequency deviations, and also eventually to converge to minimum cost allocations automatically. Thus this iterative procedure simultaneously performs the traditional functions of load frequency control and economic dispatch i.e. secondary and tertiary control.

B. Contributions

Our proposed algorithm is based on the following simple idea. If we neglect power losses, the minimum cost allocation of power is achieved when the marginal cost of an additional unit of generation is equal for all generators. Thus, when there is a positive power imbalance (i.e. instantaneous load exceeds rated generation), it is intuitively reasonable for a generator with a lower marginal cost to increase its generation by a larger amount than one with a higher marginal cost. Conversely, when there is a negative power imbalance, it is reasonable for a high marginal cost generator to reduce its generation by a relatively larger amount. On the other hand, one must also account for the fact that the generator whose cost function has a higher second derivative, will undergo a faster rise in its marginal cost for the same unit of added generation. The algorithm of this paper accordingly modifies the algorithm of [1] to accommodate this effect. As in [1], each generator has access only to its own cost function, and (local) measurements of frequency deviations which serves as an indirect measure of the power imbalance in the grid.

Our distributed approach offers the following features that makes it an interesting alternative to the traditional centralized approach for certain applications.

1) *Scalability.* Centralized dispatch algorithms require knowledge of the cost functions of each generator which limits their scalability. Our algorithm is fully distributed and thus, more scalable which makes it especially attractive for power grids supplied by a large number of small distributed generators.
2) *Dynamic adaptability.* The distributed algorithm responds automatically to changes in loads and in generation costs and modifies the power allocations accordingly. This can be attractive when the generation and loads are highly variable as in grids with a large number of intermittent alternative energy generators.
3) *Model independence.* The distributed approach solves the optimization problem in an iterative "online" fashion and as such, does not require a detailed modeling of power flows or line losses.

The main contribution of this paper is to describe a distributed algorithm for optimal dispatch of power generators, establish some of its optimality and convergence properties, illustrate its performance using simulation results and motivate further research into new techniques for the control and management of an electric grid with high penetration of alternative energy sources and advanced capabilities such as smart meters and flexible loads. Specifically, we show the following properties:

(A) For a constant load our algorithm exponentially forces the network to remove the frequency deviation, with or without power losses.
(B) Whenever there is a power imbalance, the network reallocates the power generation across generators in such a way as to reduce the difference between the marginal costs of its constituent generators.
(C) If the load remains constant, under our algorithm, the network corrects for the frequency deviations and may reach an equilibrium without necessarily equalizing the marginal costs. *However, such a stationary point, with zero frequency deviation but unequal marginal costs is an unstable stationary point.* Taken together with (B), this implies that random load fluctuations will drive the algorithm eventually to equal marginal costs.
(D) If power losses are negligible, equal marginal costs implies optimality i.e. minimization of the total generation cost. Even with losses the algorithm continues to eventually equalize the marginal costs. This of course is suboptimal in general, in terms of minimization of total generation cost. However, we demonstrate that near optimality is achieved under small losses, and quantify the extent of the suboptimality.

We also present simulation results to illustrate the behavior of the algorithm and compare its performance with traditional centralized dispatch algorithms. We also show that the algorithm still achieves near-optimal performance with losses provided the level of losses is not too large.

II. Problem description

We model the economic dispatch problem as follows. We assume that there are N generators supplying power to the network. At time-step k, we denote the total power consumed by $P_{load}[k]$, power losses in the grid by $P_{loss}[k]$, and the active power set point for generator i at the rated system frequency by $R_i[k],\ i \in 1 \dots N$. As a result, the power imbalance in the system is given by

$$\Delta P[k] = P_{load}[k] - \sum_{i=1}^{N} R_i[k] + P_{loss}[k] \qquad \text{(II.1)}$$

This model is illustrated in Fig. II. We neglect the effects of reactive power flows, voltage deviations and transients as is standard for economic dispatch problems.

Note that $R_i[k]$ represents the active power *set-point*; the actual active power produced by each generator is determined by its primary controller which uses $R_i[k]$ as a reference. More precisely, we assume that each generator is equipped with a primary controller that implements a power-frequency characteristic (see for e.g. [15]) with a *negative droop*, so that the active power $P_i[k]$ produced at time k is related to the grid

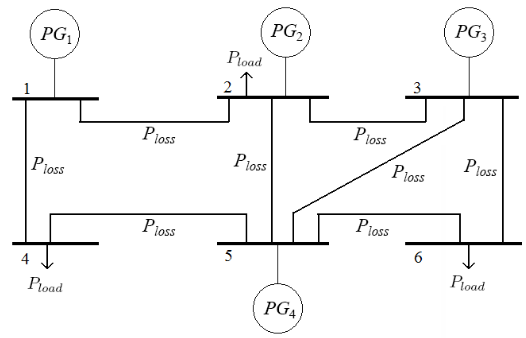

Fig. 1. Model of the electric grid for economic dispatch.

frequency $f[k]$ as:

$$P_i[k] = R_i[k] - \beta_i \Delta f[k] \tag{II.2}$$
$$\text{where } \Delta f[k] \doteq f[k] - f_0$$

Note that at the rated frequency f_0 (usually 60 Hz or 50 Hz), $P_i[k] \equiv R_i[k]$. Since there is no energy storage in the grid, conservation of energy requires that

$$P_{load}[k] - \sum_{i=1}^{N} P_i[k] + P_{loss}[k] \equiv 0.$$

Combining (II.2) and (II.1), we get

$$\begin{aligned}\Delta P[k] &= P_{load}[k] - \sum_{i=1}^{N} P_i[k] - \Delta f[k] \sum_i \beta_i + P_{loss}[k] \\ &= -\beta \Delta f[k] \end{aligned} \tag{II.3}$$

where we have denoted $\beta \doteq \sum_i \beta_i$. In other words, the total imbalance between the rated generation power and the load causes a proportional frequency deviation $\Delta f[k] = \frac{1}{\beta}\Delta P[k]$ on the grid that can be monitored continuously by each generator. This is analogous to the ACE observed by the secondary controller in a traditional LFC implementation. We assume that β remains constant for all values of $R_i[k]$ and $\Delta P[k]$. This is a reasonable assumption for small frequency deviations.

Assumption 2.1: Each $J_i(\cdot)$ is twice differentiable and a convex function. Specifically, there exist $\eta_1 > 0$ and a positive nondecreasing function $f(\cdot)$, that is finite for all finite argument, such that for all P, and $i \in \{1, \cdots, N\}$, the second derivative

$$J_i''(P) \doteq \frac{d^2 J_i(P)}{dP^2},$$

satisfies $\eta_1 \le J_i''(P) \le f(P)$. Further $J_i''(\cdot)$ is piece-wise continuous.

Let us define the marginal costs

$$J_i'(P) \doteq \frac{dJ_i(P)}{dP}.$$

We also assume non-zero idling cost:

Assumption 2.2: There exists $\eta_2 > 0$, such that $J_i'(0) > \eta_2$ for all $i \in \{1, \cdots, N\}$.

Intuitively, the above assumptions require that the cost function be monotonically increasing, convex and bounded.

Finally we assume that the power losses in the grid vary smoothly with the generator powers. We denote

$$\mathbf{R} \doteq [R_1,\ R_2,\ \ldots,\ R_N]^T \tag{II.4}$$

i.e. $\mathbf{R}(\cdot) : \mathbb{R} \to \mathbb{R}^N$ has elements representing the power set-points across the generators. Further we denote the power loss by

$$L(\mathbf{R}) \doteq P_{loss}. \tag{II.5}$$

Assumption 2.3: The function $L(\cdot)$ is nonnegative, differentiable and there holds:

$$\gamma_i(\mathbf{R}) \doteq \frac{\partial L}{\partial R_i} \le \gamma_0 < 1.$$

Thus, the γ_i, represent the fraction of an additional unit of power from generator i that is lost in the power grid.

It is easy to show using Lagrangian techniques that under Assumption 2.1, and zero power loss, the solution to the above optimization problem satisfies:

$$J_i'(R_i) \equiv \frac{dJ_i(R_i)}{dR_i} = \text{constant} \doteq \lambda, \forall i \in \{1 \ldots N\} \tag{II.6}$$

Equation (II.6) has the well known interpretation that at the minimum cost allocation of power, the marginal cost $J_i'(R_i)$ of an additional unit of power is constant across all generators. The optimal marginal cost is λ.

III. Distributed algorithm for optimal economic dispatch

We now describe our distributed algorithm. This is an iterative algorithm under which at time-step k, generator i updates its rated power as follows.

$$R_i[k+1] = \begin{cases} R_i[k] + \left(\frac{\alpha_1 \Delta P[k]}{J_i'(R_i[k]) J_i''(R_i[k])}\right), & \Delta P[k] \ge 0 \\ R_i[k] + \alpha_2 \Delta P[k] \frac{J_i'(R_i[k])}{J_i''(R_i[k])}, & \text{else} \end{cases} \tag{III.7}$$

where $\alpha_1 > 0$ and $\alpha_2 > 0$ are paramaters controlling the rate of adaptation. Note that all generators are calibrated with the same value for these parameters.

The intuition behind (III.7) is explained as follows. When the power imbalance $\Delta P[k]$ is positive, then the generators make a small increase to their rated powers in inverse proportion to their marginal cost. Thus generators with low marginal costs increase their allocation more rapidly than high cost generators. Conversely when the $\Delta P[k]$ is negative, then the low cost generator reduces its power less rapidly compared to high cost generators. The inclusion of the second derivative reflects the fact that a large second derivative causes larger changes to the marginal costs.

Observe that to implement this algorithm each generator only needs knowledge of its own cost function, in addition to a term proportional to the load imbalance that can be obtained by locally measuring the load frequency deviation.

Thus the algorithm is implemented totally locally. As will be proved in the sequel, over time, this algorithm tends to equalize the marginal costs across generators and thus leads to the minimum cost solution, when $P_{loss}[k] \equiv 0$.

We next examine the properties of the algorithm in more detail.

IV. Properties of the Distributed Dispatch Algorithm

It is more convenient to analyze this algorithm by looking at its continuous time version described as follows. The results directly extend to the discrete time version for small α_i. The continuous time algorithm is as follows:

$$\frac{dR_i(t)}{dt} = \begin{cases} \alpha_1 \Delta P(t) \left(\frac{1}{J_i'(R_i(t)) J_i''(R_i(t))} \right), & \text{if } \Delta P(t) \geq 0 \\ \alpha_2 \Delta P(t) \frac{J_i'(R_i(t))}{J_i''(R_i(t))}, & \text{otherwise} \end{cases} \tag{IV.8}$$

Denote the vector corresponding to the optimal allocation (II.6) by $\mathbf{R}_{opt}$. Further let $\mathbf{1} \doteq [1,\ 1, \ldots,\ 1]^T$ denote the "all-ones" vector. Then we can write

$$\Delta P(t) \equiv P_{load}(t) - \mathbf{1}^T \mathbf{R}(t) + L(\mathbf{R}(t)) \tag{IV.9}$$

Observe, under Assumption 2.3, with

$$\Gamma(\mathbf{R}) = [\gamma_1(\mathbf{R}), \cdots, \gamma_N(\mathbf{R})]^T, \tag{IV.10}$$

with a constant load, there hold:

$$\begin{aligned} \Delta \dot{P}(t) &= -\mathbf{1}^T \dot{\mathbf{R}}(t) + \dot{L}(\mathbf{R}(t)) \\ &= -[\mathbf{1} - \Gamma(\mathbf{R})]^T \dot{\mathbf{R}}(t). \end{aligned} \tag{IV.11}$$

Observe, *regardless of whether* $\gamma_i(\mathbf{R})$ *is negative*, because of Assumption 2.3, for all i there holds:

$$1 - \gamma_i(\mathbf{R}) \geq 1 - \gamma_0 > 0. \tag{IV.12}$$

Consequently, with a positive $\Delta P(t)$, the $\dot{R}_i$ are strictly positive, and

$$\begin{aligned} \Delta \dot{P}(t) &= -[\mathbf{1} - \Gamma(\mathbf{R})]^T \dot{\mathbf{R}}(t) \\ &\leq -(1-\gamma_0)\, \mathbf{1}^T \dot{\mathbf{R}} \\ &\leq 0. \end{aligned} \tag{IV.13}$$

Similarly, when $\Delta P(t)$ is negative, the $\dot{R}_i$ are nonpositive, and

$$\begin{aligned} \Delta \dot{P}(t) &= -[\mathbf{1} - \Gamma(\mathbf{R})]^T \dot{\mathbf{R}}(t) \\ &\geq -(1-\gamma_0)\, \mathbf{1}^T \dot{\mathbf{R}} \\ &\geq 0. \end{aligned} \tag{IV.14}$$

We next prove the existence of a solution to (IV.8).

Theorem 4.1: Consider (IV.8), under assumptions 2.1, 2.2and 2.3, with $\mathbf{R}$ as in (II.4), and $P_{load}(t)$ a positive constant. Suppose for all $i \in \{1, \cdots N\}$, $R_i(0) > 0$ and $\Delta P(0)$ is finite. Then the solution to (IV.8) exists and is unique. Further, for all $i \in \{1, \cdots, N\}$, and all $t \geq 0$, $J_i'(R_i(t)) > 0$ whenever $\Delta P(0) > 0$. On the other hand when $\Delta P(0) < 0$ for all $i \in \{1, \cdots, N\}$, and all $t \geq 0$, $J_i'(R_i(t)) \geq 0$.

We next show that (IV.8) induces $\Delta P(t)$ to converge exponentially to zero.

Theorem 4.2: Under the conditions of Theorem 4.1 suppose $P_{load}(t)$ is a positive constant. Then the power imbalance $\Delta P(t)$ converges exponentially to zero.

The Theorem does not guarantee that the equilibrium point has minimum cost in the lossless case. Indeed for a constant load, $\Delta P(t)$ may converge before the marginals are equalized. The next theorem shows that in fact the algorithm drives the marginals towards equalization *while a load imbalance persists*.

Theorem 4.3: Under the conditions of Theorem 4.1, suppose $\Delta P(t) \neq 0$, and for some i, j, $J_i'(R_i(t)) > J_j'(R_j(t))$. Then

$$\left(\frac{d(J_i'(R_i(t)) - J_j'(R_j(t)))}{dt} \right) < 0.$$

To complete our study of equalization of marginals we next present the following theorem.

Theorem 4.4: Under the conditions of Theorem 4.1, consider the stationary point where $\Delta P(t) = 0$, but for some $\{i, j\} \subset \{1, \cdots, N\}$,

$$J_i'(R_i(t)) \neq J_j'(R_j(t)).$$

Then this stationary point is unstable.

Taken together, the significance of Theorems 4.3 and 4.4, is as follows. While $\Delta P(t) \neq 0$, the algorithm will tend to drive the marginals closer. If $\Delta P(t)$, becomes zero before the marginals are equalized, then the slightest noise in the R_i or load fluctuations that enforce the condition $\Delta P(t) \neq 0$, will again tend to drive the marginals closer to each other. Over time the practical effect of this is to equalize the marginals.

Observe that all results in this section accommodate power losses. Of course equalization of the marginals is suboptimal unless the power losses are zero. Section V quantifies the level of suboptimality under small power losses.

V. Lossy Performance

Thus, with zero losses the algorithm eventually achieves optimum performance. We now quantify the level of suboptimality when γ_0 in Assumption 2.3 is nonzero but small. This is physically reasonable because, recall that the γ_i (in the sequel and here we drop the argument $\mathbf{R}$), represents the fraction of an additional unit of power that is lost in the power grid, and in any well-designed grid, we expect this to be very small e.g. $\gamma_0 < 10\%$. In this section we demonstrate that for small γ_0, the level of suboptimality is quadratic in γ_0.

We note that Theorems 4.2 and 4.3 continue to hold for the algorithm defined in (III.7) even with power losses, however, it is no longer true that the equal marginal cost allocation achieves the minimum cost. Indeed, we can show using Lagrangian methods that the minimum cost allocation $\mathbf{R}_{opt} \doteq [R_{1,opt},\ R_{2,opt},\ \ldots,\ R_{N,opt}]^T$ satisfies

$$\left(\frac{1}{(1-\gamma_i)} \frac{dJ_i}{dR_i} \right)_{R_i = R_{i,opt}} = \left(\frac{1}{(1-\gamma_j)} \frac{dJ_j}{dR_j} \right)_{R_j = R_{j,opt}}, \ \forall\ i, j \tag{V.15}$$

We next show that the resulting sub-optimality is small provided that the marginal power losses (i.e. γ_i) are not too large.

We have noted that the algorithm of (III.7) tends to equalize marginal cost allocation which may not be the same as (V.15). Let us denote this equalized marginal cost allocation by $\mathbf{R}_0 \doteq [R_{1,0},\ R_{2,0},\ \ldots,\ R_{N,0}]^T$ and the marginal cost corresponding to this allocation as

$$\lambda_0 \doteq \left(\frac{dJ_i}{dR_i}\right)_{R_i=R_{i,0}} \equiv \left(\frac{dJ_j}{dR_j}\right)_{R_j=R_{j,0}}.$$

From our assumption that the marginal power losses are small, under suitable smoothness assumptions, $\mathbf{r} \doteq \mathbf{R}_{opt} - \mathbf{R}_0$ will be small, where $\mathbf{r} \equiv [r_1,\ r_2,\ \ldots,\ r_N]^T$ is the deviation of the equal marginal cost allocation from the minimum cost allocation. Thus we can write

$$\begin{aligned}\left(\frac{dJ_i}{dR_i}\right)_{R_i=R_{i,opt}} &\approx \left(\frac{dJ_i}{dR_i}\right)_{R_i=R_{i,0}} + r_i \frac{d^2 J_i}{dR_i^2} \\ &= \lambda_0 \left(1 + \frac{r_i}{\lambda_0}\frac{d^2 J_i}{dR_i^2}\right) \qquad \text{(V.16)}\end{aligned}$$

Furthermore, since both $\mathbf{R}_0$ and $\mathbf{R}_{opt}$ satisfy the power balance constraint, we have $\mathbf{1}^T\mathbf{R}_{opt} - L(\mathbf{R}_{opt}) \equiv \mathbf{1}^T\mathbf{R}_0 - L(\mathbf{R}_0) \equiv P_{load}$. Thus we have $\mathbf{1}^T\mathbf{r} = L(\mathbf{R}_{opt}) - L(\mathbf{R}_0) \equiv \sum_i \gamma_i r_i$, which gives after rearrangement $\sum_i (1-\gamma_i) r_i = 0$. Thus, it is not possible to have either $r_i \geq 0, \forall i$ or $r_i \leq 0, \forall i$. Without loss of generality, through a relabeling of generators if need be, assume that $r_1 > 0$ and $r_2 < 0$. Using (V.15) and (V.16), we have

$$\begin{aligned}\frac{1}{(1-\gamma_1)}\lambda_0\left(1 + \frac{r_1}{\lambda_0}\frac{d^2 J_1}{dR_1^2}\right) &\approx \frac{1}{(1-\gamma_2)}\lambda_0\left(1 + \frac{r_2}{\lambda_0}\frac{d^2 J_2}{dR_2^2}\right) \\ &\leq \frac{\lambda_0}{(1-\gamma_0)} \qquad \text{(V.17)}\end{aligned}$$

where we have used the fact that $\gamma_2 \leq \gamma_0$ and the assumption that $r_2 < 0$. Thus using the fact that $\gamma_1 < \gamma_0$, there holds:

$$r_1 \leq \frac{\lambda_0\gamma_0}{(1-\gamma_0)}\frac{1}{\frac{d^2 J_1}{dR_1^2}} \leq \frac{\lambda_0\gamma_0}{\eta_1(1-\gamma_0)} \qquad \text{(V.18)}$$

Using the same argument for all generators i, we have

$$|r_i| \leq \frac{\gamma_0}{(1-\gamma_0)}\frac{\lambda_0}{\eta_1} \qquad \text{(V.19)}$$

Finally we have

$$\begin{aligned}J(\mathbf{R}_{opt}) &\equiv \sum_i J_i(R_{i,opt}) \\ &\approx J(\mathbf{R}_0) + \sum_i r_i \left(\frac{dJ_i}{dR_{i,0}}\right) \\ &= J(\mathbf{R}_0) + \lambda_0 \sum_i r_i \\ &= J(\mathbf{R}_0) + \lambda_0 \left(L(\mathbf{R}_0) - L(\mathbf{R}_{opt})\right) \qquad \text{(V.20)} \\ &\geq J(\mathbf{R}_0) - \lambda_0\gamma_0 \sum_i |r_i| \\ &\geq J(\mathbf{R}_0) - \frac{N\gamma_0^2}{(1-\gamma_0)}\frac{\lambda_0^2}{\eta_1} \qquad \text{(V.21)}\end{aligned}$$

Equation (V.21) shows that the suboptimality of the equal marginal cost solution varies with γ_0^2 and therefore is small provided the power loss is not too large. This observation is verified by numerical simulation in Section VI.

VI. Simulation Results and gain selection

We now present numerical results to provide some intuition into the performance of the algorithm. The α_i were selected in these simulations using all the considerations note above barring the ones associated with network and generator dynamics. We consider the dispatch problem considered in [13] (Example 7.4) of allocating 1800 kW of total generation in a small grid with 6 generators. Each generator has a quadratic cost function i.e. generator i has a cost given by $J_i(R) \equiv c_i R^2 + b_i R + a_i$, where $c_i > 0$ guarantees the convexity of the cost function.

We simulated this system starting from an arbitrarily chosen, suboptimal allocation among the generators. The total load power fluctuates ramdomly around an average value of 1800 kW; these fluctuations cause power imbalances, and the resulting frequency deviations are used by the individual generators to adjust their power set-points according to (III.7). Note that the load fluctuations are essential for creating the temporary frequency deviations that drive the overall allocation towards its optimum values; however, in practice, the same effect can be obtained by deliberately introducing random fluctuations into the generation power itself if desired. The iteration time for the algorithm is taken to be 10 seconds. In this case $\alpha_1 = 0.0215$ and $\alpha_2 = 0.000075$.

Figure 2 shows the individual generation powers as well as the total cost as a function of time according to the distributed algorithm. Also shown is the minimum cost obtained by a numerical method based on neural network computing in [13]. We note that the algorithm matches within a few minutes the cost achieved by the centralized method in [13] (which was also verified by comparing with the lambda iteration method).

Figure 3 shows another example of a dispatch problem, this time there are power losses in the grid that amounts to approximately $1-2\%$ of the total generated power. This problem is solved using a numerical particle-swarm optimization technique in [14]. While our distributed algorithm does not specifically account for power losses, we expect from the analysis of Section V that for the small levels of losses,

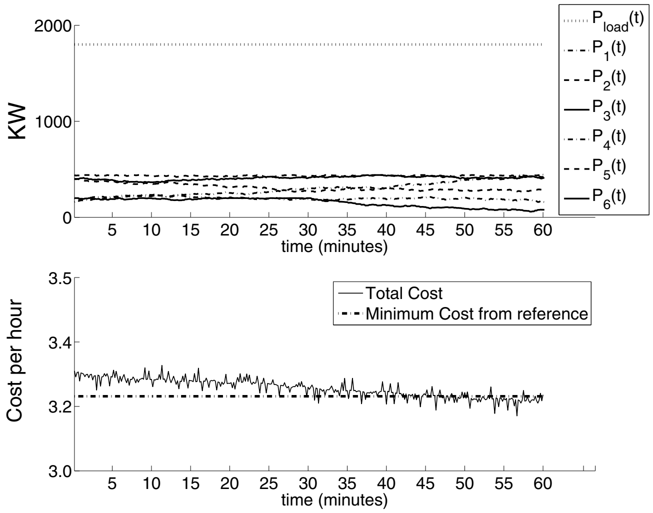

Fig. 2. Performance of the distributed dispatch algorithm.

the algorithm should achieve near-optimal performance, and we can indeed see from Fig. 3 that it eventually matches the minimum cost achieved by the centralized optimization algorithm in [14]. In this case, $\alpha_1 = 0.06$ and $\alpha_2 = 0.001$.

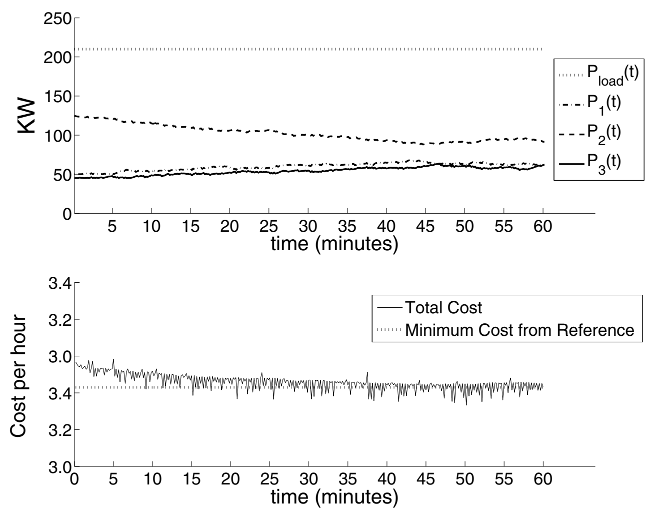

Fig. 3. The distributed dispatch algorithm in a network with power losses.

VII. Conclusion

We proposed a new distributed approach to economic dispatch, where each generator independently adjusts its power in response to frequency deviations on the grid, and we showed that it is possible to achieve the minimum cost allocation using such a method. We demonstrated through analysis and numerical simulations the effectiveness of the distributed approach and argued that it is especially attractive to an electric grid with distributed generation, smart metering capabilities and alternative energy sources.

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