Source Localization in Wireless Sensor Networks From Signal Time-of-Arrival Measurements

Enyang Xu, *Student Member, IEEE*, Zhi Ding, *Fellow, IEEE*, and Soura Dasgupta, *Fellow, IEEE*

***Abstract*—Recent advances in wireless sensor networks have led to renewed interests in the problem of source localization. Source localization has broad range of applications such as emergency rescue, asset inventory, and resource management. Among various measurement models, one important and practical source signal measurement is the received signal time of arrival (TOA) at a group of collaborative wireless sensors. Without time-stamp at the transmitter, in traditional approaches, these received TOA measurements are subtracted pairwise to form time-difference of arrival (TDOA) data for source localization, thereby leading to a 3-dB loss in signal-to-noise ratio (SNR). We take a different approach by directly applying the original measurement model without the subtraction preprocessing. We present two new methods that utilize semidefinite programming (SDP) relaxation for direct source localization. We further address the issue of robust estimation given measurement errors and inaccuracy in the locations of receiving sensors. Our results demonstrate some potential advantages of source localization based on the direct TOA data over time-difference preprocessing.**

***Index Terms*— Semidefinite programming relaxation, source localization, time of arrival.**

I. Introduction

RECENT years have witnessed tremendous growth in both interests and applications of wireless sensor networks. Among a plethora of research thrusts, one problem that has gathered substantial attention is the localization of signal emitters from signal measurements obtained at a network of collaborative and distributed signal sensors [1], [2]. We recognize that wireless source localization has broad applications, including target tracking, signal routing, interference alignment, wireless security, and emergency response. The basic setup of distributed wireless source localization involves estimating positions of signal emitters by jointly utilizing signal measurement from a subset of distributed network sensors. These sensors collaborate by sending their measurement data to a signal processing center which subsequently estimates the source location(s) according to the received measurement data.

Manuscript received June 01, 2010; revised September 26, 2010, December 16, 2010, and January 28, 2011; accepted January 29, 2011. Date of publication February 17, 2011; date of current version May 18, 2011. The material is based on works supported by the National Science Foundation Grants CCF-0830706, CCF-0803747 and CCF-0729025. This work has been presented in part at the IEEE International Conference on Acoustics, Speech and Signal Processing (IEEE ICASSP), Dallas, Texas, March 14–19, 2010.

E. Xu and Z. Ding are with the Department of Electrical and Computer Engineering, University of California, Davis, CA 95616 USA (e-mail: eyxu@ece.ucdavis.edu; zding@ece.ucdavis.edu).

S. Dasgupta is with the Department of Electrical and Computer Engineering, University of Iowa, Iowa City, IA 52242 USA (e-mail: dasgupta@engineering.uiowa.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSP.2011.2116012

As a well-studied problem in sensor networks, there exist various established methods for source localization that are based on measurement models of received signal time of arrival (TOA), distance measurement, received signal strength (RSS), signal angle of arrival (AOA), and their combinations. The sensors should know and utilize some features of the signal from the unknown emitter in order to obtain these measurements at the receiver [2]. In many radio signal applications, distance information is not directly available and must be estimated based on signal measurement such as strength and time of arrival. On the other hand, received signal strength measurements can also be very sensitive to the channel environment. For example, in an environment with rich scatters, signal strength measurement can be difficult to model and relate to the source location information. For these reasons, other measurement models may be more practical. In this work, we are particularly interested in the simple model based on received signals' time of arrival measurement. In the TOA model, each sensor only needs to identify a special signal feature such as a known preamble to record its arrival time. Based on the model that relates the TOA to the source-sensor location information, we can directly estimate the source location from multiple TOA measurements.

In most radio environments with direct line-of-sight path or with scatters close to the source or sensor, the TOA measurement is directly correlated to the distance between the source and the sensor as the radio propagation velocity is well known. One practical obstacle is the typical lack of synchronization between the source and the receiver. In other words, the receivers often are not aware of the precise starting time instant of source transmission t_0. The uncertainty with respect to the starting time of transmission instant t_0 causes a common time offset among all the received TOA measurements, which can potentially lead to significant localization error. For this reason, several existing works assume source-sensor synchronization [3], [4] so that t_0 is known. However, this knowledge requires the cooperation between the source and the sensors, an assumption that severely limits the practical application of such algorithms. A very popular alternative in the literature to deal with the unknown t_0 is to preprocess the TOA measurement by utilizing only the difference of TOA measurements from various sensors. The preprocessing of subtracting pairwise TOA measurement removes the unknown t_0 from the measurement information and simplifies the source localization problem into the time-difference of arrival (TDOA) model.

The TDOA model considers the difference of the arrive time between the clock-synchronized sensor nodes. However, the subtraction of pairwise TOA measurements leads to correlated noise in TDOA [2], and more importantly, strengthens the measurement noise by 3 dB. Despite the drawbacks, the simpler TDOA model has spurred a number of effective methods

1053-587X/$26.00 © 2011 IEEE

designed for TDOA measurements in source localization. Various solutions range from linear [2], nonlinear [5], and convex optimization [6], [7] approaches. In fact, some TDOA works view TDOA as the original (noisy) measurement and neglect the subtraction step. As a result, the actual effect of the TDOA preprocessing on the localization tends to be blurred.

Recently, convex optimization techniques have been applied in source localization. These optimization techniques can be grouped into two categories: seconder order cone programming (SOCP) and semidefinite programming (SDP). Both categories apply various types of relaxation methods to the original problem to arrive at convex SOCP and SDP problems. In [8], the distance constraints are relaxed and the problem is formulated as SOCP. The SDP approach has appeared in different measurement models including distance model [9], TOA model [10], and TDOA model [6], [7]. Note, however, that the TOA model of [10] requires sensors to have the knowledge of the source signal starting transmission time instant t_0. This synchronization requirement renders the TOA model of [10] less general and cannot be applied to problems without source-sensor cooperation and synchronization. For the practical TOA model without source-sensor synchronization, the unknown starting time of source signal transmission further complicates the localization problem. To the best of our knowledge, there exists no work in the literature that solve the more general TOA problem directly via convex optimization.

In this work, we apply the original TOA measurement model for source localization. Unlike existing methods that either assume t_0 to be known or use TOA subtraction, our approach makes no such assumption or preprocessing. Our goal is to present practical algorithms while avoiding the unnecessary noise enhancement and noise coloring associated with the TDOA model. Our contributions are as follows. In our first work, we propose a two-step approach for TOA that begins by estimating the time of transmission t_0. The two-step approach yields a SDP algorithm that can approximate a maximum likelihood estimate of the source location. We also present a second SDP approach for source localization based on minimizing the maximum error measurement between the observed propagation time and the modeled propagation time. Both methods are shown to be effective without TDOA preprocessing in estimating source locations. Furthermore, we investigate the robustness issues that arise because of inaccuracies in the sensor locations. We develop a robust TOA localization algorithm to tackle the robustness problem due to such sensor location errors.

II. Problem Statement

A. A More General and Practical Time of Arrival Model

We first describe the practical TOA model for source localization. Consider a network of N distributed sensors at the positions denoted by a set of m-dimensional vectors $\mathbf{x}_1, \ldots, \mathbf{x}_N$ (with $m = 2$ or 3 for 2-dimensional or 3-dimensional localization, respectively). These sensors cooperate by helping a data fusion center (DFC) determine an unknown source location denoted by an$m$-dimensional vector $\mathbf{y}$. Note that we focus only on a propagation environment in which a line-of-sight (LOS) path exists or in which nearby scatters around the source and the sensor can provide a near-LOS path. In other words, all the time of arrival measurements can be approximately obtained from the LOS path. By collecting measurements from the sensors, a data fusion center attempts to estimate $\hat{\mathbf{y}}$ of the source location.

During the localization process, each sensor detects the time of arrival measurement of the source signal at its receiver based on particular signal features (e.g., preamble) transmitted by the source node. Given an LOS propagation path, the time of arrival measurement t_i at sensor node $\mathbf{x}_i$ can be easily modeled as

$$t_i = \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| + t_0 + n_i, \qquad i = 1, 2, \cdots, N \tag{1}$$

where c is the speed of light, $\|\cdot\|$ denotes the Euclidean norm, t_0 is the unknown time instant at which the source transmits the signal to be measured, and n_i is the additive measurement noise (error) with zero mean. We note that the sensors only estimate the signal TOA t_i instead of the signal propagation time $\|\mathbf{x}_i - \mathbf{y}\|/c$. In order to estimate the propagation time, the source must cooperate by synchronizing its signal "time of transmission" t_0 with the sensors, or it must encode a time stamp within the transmitted signal to inform the sensors what t_0 is. Without such time synchronization or time stamp, the TOA measurement consists of an additional unknown t_0. In some existing approaches, the resulting TOA measurement are preprocessed through pairwise subtraction to generate the measurement for time difference of arrival based localization [6], independent of t_0.

Without any other prior assumptions on the statistics of the TOA measurements, a least square (LS) estimator can be used for the source localization problem, i.e.,

$$(\hat{\mathbf{y}}, \hat{t_0}) = \arg\min_{\mathbf{y}, t_0} \sum_{i=1}^{N} \left(t_i - \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| - t_0 \right)^2. \tag{2}$$

Using brute force, we can implement direct optimization by searching for the optimum $\mathbf{y}$ and t_0 that minimize (2). This criterion is maximum likelihood (ML) for uncorrelated Gaussian measurement noises $\{n_i\}$. Because this TOA model needs to estimate both $\mathbf{y}$ and t_0 jointly, the ML optimization problem can be rather challenging as a multidimensional search problem. In particular, the brute force LS criterion of (2) is a nonconvex problem potentially admitting multiple local minima. Existing algorithms achieved only limited successes, even for small problem sizes [11].

B. TOA Model Versus TDOA Model

Recognizing that the unknown t_0 is not of direct interest in source localization, one common alternative to solving the joint estimation problem is to use the pairwise difference of the arrival times among the sensor nodes. In order to obtain the time-difference of arrival, a simple preprocessing of the TOA measurement is implemented by

$$\Delta_{ij} = t_i - t_j = \frac{1}{c}\left(\|\mathbf{x}_i - \mathbf{y}\| - \|\mathbf{x}_j - \mathbf{y}\|\right) + \underbrace{n_i - n_j}_{v_{ij}}, \tag{3}$$

where Δ_{ij} is the TDOA measurement.

After the preprocessing, the unknown parameter t_0 is removed. However, there are two problems for this processing. Firstly, we note that the noise terms v_{ij} in (3) are no longer independent. For example, the noises v_{ij} and v_{kj} are correlated since they have n_j in common. In addition, in comparison with the independent noise in the original TOA model (1), the subtraction also strengthens the noise in TDOA by exactly 3 dB and leads to noise correlation. For this reason, the preprocessing for obtaining TDOA may lead to performance degradation which should be avoided.

C. Cramér–Rao Lower Bound for TOA-Based Estimate

Given the TOA measurement model, the performance of any unbiased estimate of $\mathbf{y}$ would be limited by the Cramér–Rao lower bound (CRLB). The analysis of the CRLB with the unknown t_0 is equivalent to the case of known transmission time with time synchronization errors in [12]. To determine the CRLB under the general TOA measurement model, we assume that the measurement noises in (1) are independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance σ^2. Under the i.i.d. Gaussian noise assumption, the joint conditional probability density function of the measurement data $\{t_i\}$ follows:

$$p(t_1, t_2, \ldots, t_N|\mathbf{y}, t_0) = (2\pi\sigma^2)^{-N/2} \times \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(t_i - \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| - t_0\right)^2\right). \quad (4)$$

Let y_k, x_{ik} denote the k^{th} element of vectors $\mathbf{y}$, $\mathbf{x}_i$, respectively. Define $\mathbf{Z} = [y_1, \ldots, y_m, t_0]^T$ as the vector of all unknowns. The corresponding log-likelihood function (ignoring the constant term) is given by

$$L(\mathbf{Z}) = -\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(t_i - \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| - t_0\right)^2. \quad (5)$$

Then similar to [12], we can calculate each element of the Fisher information matrix $\mathbf{F}_a$. For $1 \le k \le l \le m$, we have the (k, l)th element of $\mathbf{F}_a$ as

$$[\mathbf{F}_a]_{kl} = -E\left(\frac{\partial^2}{\partial z_k \partial z_l} L(\mathbf{Z})\right) = \frac{1}{c^2\sigma^2}\sum_{i=1}^{N}\frac{(y_k - x_{ik})(y_j - x_{il})}{\|\mathbf{x}_i - \mathbf{y}\|^2}. \quad (6)$$

Additionally, for $1 \le k \le m$, we have

$$\begin{aligned}[\mathbf{F}_a]_{k(m+1)} &= [\mathbf{F}_a]_{(m+1)k} \\ &= -E\left(\frac{\partial^2}{\partial z_k \partial z_{m+1}} \ln p(t_1, \ldots, t_N|z)\right) \\ &= \frac{1}{c\sigma^2}\sum_{i=1}^{N}\frac{y_k - x_{ik}}{\|\mathbf{x}_i - \mathbf{y}\|} \end{aligned} \quad (7)$$

and

$$[\mathbf{F}_a]_{(m+1)(m+1)} = -E\left(\frac{\partial^2}{\partial z_{m+1}\partial z_{m+1}} L(\mathbf{Z})\right) = \frac{N}{\sigma^2}. \quad (8)$$

As a result, the CRLB of any unbiased estimate $\hat{\mathbf{y}}$ is

$$\text{MSE}_a = E\left(\|\hat{\mathbf{y}} - \mathbf{y}\|^2\right) \ge \sum_{i=1}^{m}\left[\mathbf{F}_a^{-1}\right]_{ii}. \quad (9)$$

However, because t_i is not linearly related with $\mathbf{y}$ in (1), we can cite the result in [13] to conclude that there exists no efficient unbiased estimate for $\mathbf{y}$. Indeed, the MLE is not efficient and no unbiased estimate can achieve the CRLB under the TOA model. Similarly, there exists no efficient unbiased estimator from the TDOA measurement [14]. Therefore, the CRLB can only serve as a benchmark when evaluating the performance of various estimates. The fact that a large gap may exist between the CRLB and the performance of a given algorithm does not invalidate the algorithm in question.

III. TOA-Based New Localization Algorithms

As noted earlier, the least square solution of (2) is a nonlinear nonconvex problem. With the potential for multiple local minima, depending on the locations of the source and the sensors, solving for its global minimum can be a serious challenge. Additionally, the lack of efficient unbiased estimate for the source localization based on TOA measurement means that the maximum-likelihood estimate (MLE) is not automatically favored. In fact some biased estimates may potentially be more accurate than unbiased ones. These facts motivate us to seek alternative, non-maximum-likelihood algorithms in TOA-based source localization.

In this section, we will develop two new TOA algorithms. One is a two step LS method, the other is based on a min-max criterion. Both algorithms utilize semidefinite relaxation to transform nonconvex problems into convex ones in order to make it easier to locate the global optimum of the original underlying problem. We now give the specifics below.

A. A New 2-Step Least Square (2LS) Formulation

Our first algorithm relies on a two-step approach. First, note that the LS estimate of $\mathbf{y}$ requires a joint optimization of both unknowns $\mathbf{y}$ and t_0. Instead of finding the $\mathbf{y}$ and t_0 jointly, we can solve for the optimum estimates by reducing the joint minimization into two steps.

First, we find the optimum transmission time t_0 as a dependent function of the unknown $\mathbf{y}$. In particular, for zero mean noise n_i in the signal model of (1), the least square estimate of the transmission time t_0 is simply

$$\hat{t}_0 = \frac{1}{N}\sum_{i=1}^{N}\left(t_i - \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\|\right). \quad (10)$$

We can now substitute t_0 with $\hat{t}_0$ in the objective function (2) in order to find the optimum source location $\mathbf{y}$ that minimizes the overall LS objective function

$$\sum_{i=1}^{N}\left[t_i - \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| - \frac{1}{N}\sum_{j=1}^{N}\left(t_j - \frac{1}{c}\|\mathbf{x}_j - \mathbf{y}\|\right)\right]^2. \quad (11)$$

The resulting objective function is nonconvex and should be solved with well-behaved algorithms.

Next, we can applied similar relaxation techniques in [6]. To derive a convex optimization relaxation, we introduce auxiliary variables

$$\tau_i = \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\|, \quad 1 \le i \le N.$$

Let us denote $t = [t_1, \ldots, t_N]^T$, $\tau = [\tau_1, \ldots, \tau_N]^T$, $\mathbf{Q} = \tau\tau^T$, $G = I - \mathbf{1} \cdot \mathbf{1}^T$, where $\mathbf{1}^T = [1\ 1 \cdots 1]_{1\times N}$. With these notations, we can rewrite the objective function of (11) for minimization as

$$\text{Tr}\left[G^T G(\mathbf{Q} - 2t\tau^T + tt^T)\right] \quad (12)$$

where Tr(.) represents the trace of a matrix.

Notice that the problem described in (12) under the constraints $\tau_i = \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\|$ and $\mathbf{Q} = \tau\tau^T$ is identical to the original optimization problem and is nonconvex. Clearly, this objective function form is a linear function of both $\mathbf{Q}$ and τ and is convex. However, because of the constraints $\tau_i = \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\|$ and $\mathbf{Q} = \tau\tau^T$ are nonconvex, the solution remains difficult. Our next task is to relax the nonconvex constraints into convex constraints that remain tightly connected with the original constraints.

To begin, considering the auxiliary variables τ_i, we need to enforce the constraint $\tau_i = \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\|$. It is helpful to realize that

$$(\mathbf{x}_i - \mathbf{y})^T(\mathbf{x}_j - \mathbf{y}) = \begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \begin{bmatrix} \mathbf{x}_j \\ -1 \end{bmatrix}$$

in which $y_s = \mathbf{y}^T\mathbf{y}$. Therefore, utilizing the matrix notation $\mathbf{Q}$, this constraint can be written as

$$\begin{aligned} \mathbf{Q}_{ii} &= \tau_i^2 = \frac{1}{c^2}(\mathbf{x}_i - \mathbf{y})^T(\mathbf{x}_i - \mathbf{y}) \\ &= \frac{1}{c^2}\begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}. \end{aligned} \tag{13}$$

This constraint is now convex in terms of variables $\mathbf{Q}$, $\mathbf{y}$, and y_s. Given the variable matrix $\mathbf{Q}$, we can also apply Cauchy–Schwartz inequality to yield

$$\|\mathbf{x}_i - \mathbf{y}\|\|\mathbf{x}_j - \mathbf{y}\| \ge \left|(\mathbf{x}_i - \mathbf{y})^T(\mathbf{x}_j - \mathbf{y})\right|.$$

This constraint inequality can be written as

$$\begin{aligned} \mathbf{Q}_{ij} &= \tau_i\tau_j = \frac{1}{c^2}\|\mathbf{x}_i - \mathbf{y}\|\|\mathbf{x}_j - \mathbf{y}\| \\ &\ge \frac{1}{c^2}\left|\begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \begin{bmatrix} \mathbf{x}_j \\ -1 \end{bmatrix}\right| \end{aligned} \tag{14}$$

which is also convex in terms of $\mathbf{Q}$, $\mathbf{y}$, and y_s.

We now still have two nonlinear and nonconvex constraints in the form of equalities$\mathbf{Q} = \tau\tau^T$ and $y_s = \mathbf{y}^T\mathbf{y}$. We apply semidefinite relaxation such that they are relaxed into convex inequalities $\mathbf{Q} \succeq \tau\tau^T$ and $y_s \succeq \mathbf{y}^T\mathbf{y}$. Furthermore, they can be written as linear matrix inequalities (LMI):

$$\begin{bmatrix} \mathbf{Q} & \tau \\ \tau^T & 1 \end{bmatrix} \succeq 0, \quad \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \succeq 0. \tag{15}$$

We now have transformed the LS problem into a convex optimization problem:

$$\begin{aligned} &\min_{\mathbf{y}, y_s, \tau, \mathbf{Q}} \operatorname{Tr}\left[G^TG(\mathbf{Q} - 2t\tau^T + tt^T)\right] \\ &\text{s.t.} \begin{bmatrix} \mathbf{Q} & \tau \\ \tau^T & 1 \end{bmatrix} \succeq 0, \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \succeq 0, \\ &\quad \mathbf{Q}_{ii} = \frac{1}{c^2}\begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}, \\ &\quad \mathbf{Q}_{ij} \ge \frac{1}{c^2}\left|\begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \begin{bmatrix} \mathbf{x}_j \\ -1 \end{bmatrix}\right|. \end{aligned} \tag{16}$$

We note, however, that this simplistic convex optimization formulation is still prone to ambiguities. For example, the value of the LS function (2) would not change when $\frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\|$ increases and t_0 decreases or$\frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\|$ decreases and t_0 increases. Therefore, we need to add a penalty term here to avoid the ambiguity. In other words, we introduce an extra penalty $\eta\sum_{i=1}^N\sum_{j=1}^N \mathbf{Q}_{ij}$ into the objective function where $\eta > 0$ is a penalty factor.

Finally, we recast the constrained minimization problem into an SDP form of

$$\begin{aligned} &\min_{\mathbf{y}, y_s, \tau, \mathbf{Q}} \operatorname{Tr}\left[G^TG(\mathbf{Q} - 2t\tau^T + tt^T)\right] + \eta\sum_{i=1}^N\sum_{j=1}^N \mathbf{Q}_{ij} \\ &\text{s.t.} \begin{bmatrix} \mathbf{Q} & \tau \\ \tau^T & 1 \end{bmatrix} \succeq 0, \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \succeq 0, \\ &\quad \mathbf{Q}_{ii} = \frac{1}{c^2}\begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}, \\ &\quad \mathbf{Q}_{ij} \ge \frac{1}{c^2}\left|\begin{bmatrix} \mathbf{x}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \begin{bmatrix} \mathbf{x}_j \\ -1 \end{bmatrix}\right|, \\ &\quad i = 1, \cdots, N, j = i+1, \cdots, N. \end{aligned} \tag{17}$$

The convex optimization problem of (17) can be solved efficiently using interior point methods [15]. In this paper, we apply the popular SDP solvers SeDuMi [16] to numerically solve the problem in our tests and simulations.

We note that in the SDP formulation, a suitable selection of η is needed to achieve good solutions. Heuristically, the weighting factor should be related to the distance between the sensor and the source nodes. Therefore, we propose to determine the value of η proportional to the average TOA measurement

$$\eta = \alpha \times \frac{1}{N}\sum_{i=1}^N t_i. \tag{18}$$

The suitable value of α will be discussed later when we present the simulation results.

B. Min-Max Formulation Under Unknown Noise Characteristics

The LS formulation is optimum in the maximum likelihood sense when the TOA measurement noise is assumed to be i.i.d. Gaussian. In practice, however, TOA measurement noise may exhibit different characteristics. Therefore, there is strong incentive for us to develop effective localization algorithms that are less dependent of noise assumptions.

Steering away from the LS objective function, we can rewrite the TOA measurement of (1) into

$$t_i - t_0 = \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| + n_i. \tag{19}$$

Squaring in both sides, we get

$$(t_i - t_0)^2 - \frac{1}{c^2}\|\mathbf{x}_i - \mathbf{y}\|^2 = \underbrace{\left(\frac{2}{c}\|\mathbf{x}_i - \mathbf{y}\| + n_i\right)n_i}_{\omega_i}, \tag{20}$$

for $i = 1, \cdots, N$. The right-hand side of (20) is a noise term ω_i that is not independent for different indexes i. At modest to high SNR, $\frac{2}{c}\|\mathbf{x}_i - \mathbf{y}\|$ dominates n_i and hence $\omega_i \approx \frac{2}{c}\|\mathbf{x}_i - \mathbf{y}\|n_i$.

One way to estimate the optimum $\mathbf{y}$ without assuming any particular characteristics on ω_i is to minimize the ℓ_∞ norm of ω_i. This approach makes no assumption on the noise distribution or on the noise correlation. It simply tries to minimize the peak error. Therefore, its performance is expected to be less sensitive to the noise distribution or correlation. Thus, we propose to adopt the min-max criterion for location estimation via

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y},t_0} \max_{i=1,\cdots,N} \left|(t_i - t_0)^2 - \frac{1}{c^2}\|\mathbf{x}_i - \mathbf{y}\|^2\right|. \quad (21)$$

Note again that this min-max formulation (21) is a nonconvex problem. Nevertheless, it is quite amenable to semidefinite relaxations as shown below.

First, let us introduce two auxiliary variables $y_s = \mathbf{y}^T\mathbf{y}$ and $t_s = t_0^2$. They allow us to rewrite (21) as

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y},y_s,t_0,t_s} \max_{i=1,\cdots,N} \left| t_s - 2t_it_0 + t_i^2 - \frac{1}{c^2}\left(y_s - 2\mathbf{x}_i^T\mathbf{y} + \mathbf{x}_i^T\mathbf{x}_i\right)\right| \quad (22)$$

which is a convex function in terms of variables $\mathbf{y}$, y_s, t_0, and t_s. However, the two equality constraints $y_s = \mathbf{y}^T\mathbf{y}$ and $t_s = t_0^2$ are not convex and need to be relaxed into approximate convex constraints. In order to transform the problem formulation into a convex optimization problem, we introduce two convex relaxations on the equality constraints. Specifically, we relax the two equalities $y_s = \mathbf{y}^T\mathbf{y}$ and $t_s = t_0^2$ into inequalities $y_s \succeq \mathbf{y}^T\mathbf{y}$ and $t_s \succeq t_0^2$, respectively. Both inequalities can be conveniently expressed in terms of linear matrix inequalities:

$$\begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \succeq 0, \begin{bmatrix} 1 & t_0 \\ t_0 & t_s \end{bmatrix} \succeq 0. \quad (23)$$

To summarize, the min-max TOA estimation criterion can be relaxed into a SDP convex optimization problem:

$$\begin{aligned} &\min_{\mathbf{y},y_s,t_0,t_s} \theta \\ \text{s.t. } & -\theta \le t_s - 2t_it_0 + t_i^2 - \frac{1}{c^2}\left(y_s - 2\mathbf{x}_i^T\mathbf{y} + \mathbf{x}_i^T\mathbf{x}_i\right) \le \theta, \\ & i = 1,\cdots,N, \\ & \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \succeq 0, \begin{bmatrix} 1 & t_0 \\ t_0 & t_s \end{bmatrix} \succeq 0. \end{aligned} \quad (24)$$

Similarly, the optimal solution of the min-max algorithm (MMA) in (24) can be found using interior point methods such as SeDuMi [16].

C. Comparisons

When comparing the 2-step least square (2LS) algorithm and the min-max algorithm (MMA) for source localization in TOA models, it is clear that the MMA has lower computation complexity. Additionally, the MMA does not require the selection of tuning parameter η and therefore easier to use. On the other hand, because of the measurement processing by MMA in squaring the measurement t_i^2, the resulting noise enhancement may lead to some performance loss. The complexity tradeoff and performance difference between the two algorithms will be shown later in our simulation results.

IV. Robust Localization Under Sensor Location Errors

In preceding development of the 2LS and MMA source localization algorithms, we have made the assumption that the network knowledge of the sensor locations is accurate. In other words, $\mathbf{x}_i$ is accurately known. We should consider the cases in practice when such knowledge may not be exact because of imperfections in sensor deployment, positioning, and delay of position updating. In fact, it is often difficult to obtain precise locations of the sensor nodes in sensor networks. We are interested in making source localization more robust under such information uncertainties. In this section, to address the problem of sensor location errors, we focus on developing robust localization methods for source localization that can accommodate inaccurate sensor locations.

A. Modeling Sensor Location Uncertainty

To model the sensor location uncertainty, let $\tilde{\mathbf{x}}_i = \mathbf{x}_i + \xi_i$ denotes the known location of the ith anchor node in which $\mathbf{x}_i$ is the actual sensor location whereas ξ_i is the location error bounded by $\|\xi_i\| \le \epsilon$. We can apply the first-order Taylor approximation to $\|\mathbf{x}_i - \mathbf{y}\|$ on $\tilde{\mathbf{x}}_i$to obtain

$$\|\mathbf{x}_i - \mathbf{y}\| \approx \|\tilde{\mathbf{x}}_i - \mathbf{y}\| - \frac{\xi_i^T(\tilde{\mathbf{x}}_i - \mathbf{y})}{\|\tilde{\mathbf{x}}_i - \mathbf{y}\|} + o(\|\xi_i\|). \quad (25)$$

Substituting the approximation of (25) into (1), the TOA measurement can be approximated by

$$t_i = \frac{1}{c}\|\tilde{\mathbf{x}}_i - \mathbf{y}\| - \frac{\xi_i^T(\tilde{\mathbf{x}}_i - \mathbf{y})}{c\|\tilde{\mathbf{x}}_i - \mathbf{y}\|} + t_0 + n_i + o(\|\xi_i\|). \quad (26)$$

Once again, since $\mathbf{y}$ is not linearly related to t_i in (25), there is no efficient unbiased estimator in this case according to [13].

For convenience, denote $\delta = [\delta_1, \ldots, \delta_N]^T$ in which $\delta_i = \frac{\xi_i^T(\tilde{\mathbf{x}}_i - \mathbf{y})}{c\|\tilde{\mathbf{x}}_i - \mathbf{y}\|}$. Because

$$\begin{aligned} \|\delta_i\|^2 &= \frac{\xi_i^T(\tilde{\mathbf{x}}_i - \mathbf{y})\xi_i^T(\tilde{\mathbf{x}}_i - \mathbf{y})}{c^2\|\tilde{\mathbf{x}}_i - \mathbf{y}\|^2} \\ &\le \frac{\xi_i^T\xi_i(\tilde{\mathbf{x}}_i - \mathbf{y})^T(\tilde{\mathbf{x}}_i - \mathbf{y})}{c^2\|\tilde{\mathbf{x}}_i - \mathbf{y}\|^2} = \frac{\xi_i^T\xi_i}{c^2}, \end{aligned} \quad (27)$$

we have constraints $\|\delta_i\| \le \epsilon/c$, for $i = 1, 2, \ldots, N$. This box constraint can also be further relaxed into the ellipsoid constraint $\|\delta\| \le \epsilon\sqrt{N}/c$. Under the constraints of sensor location uncertainty, we now modify the 2LS and MMA algorithms to take into account the sensor location errors.

B. Robust 2LS Formulation

By neglecting the high order terms in (26), we can derive a least square based formulation for localization with inaccurate position anchor nodes. Our objective is to minimize the LS formulation under a constraint on the location uncertainty δ. In particular, under the ellipsoid error constraint, our problem can be formulated into

$$\min_{\mathbf{y},t_0} \max_{\|\delta\| \le \epsilon\sqrt{N}/c} \sum_{i=1}^{N} \left(t_i - t_0 - \frac{1}{c}\|\tilde{\mathbf{x}}_i - \mathbf{y}\| + \delta_i\right)^2. \quad (28)$$

Similarly to the original 2LS formulation in the previous section, we first estimate t_0 from

$$\hat{t}_0 = \frac{1}{N}\sum_{i=1}^{N}\left(t_i - \frac{1}{c}\|\tilde{\mathbf{x}}_i - \mathbf{y}\| + \delta_i\right). \tag{29}$$

Define $\tilde{\tau}_i = \frac{1}{c}\|\tilde{\mathbf{x}}_i - \mathbf{y}\|$, $\tilde{\tau} = [\tilde{\tau}_1, \ldots, \tilde{\tau}_N]^T$, $t = [t_1, \ldots, t_N]^T$, $\tilde{\mathbf{Q}} = \tilde{\tau}\tilde{\tau}^T$, by substituting τ with $\tilde{\tau} - \delta$ in (12), we can rewrite the LS objective function (28) as

$$\begin{aligned}\mathrm{Tr}&\left[G^TG(\tilde{\mathbf{Q}} - 2t\tilde{\tau}^T + tt^T - 2\tilde{\tau}\delta^T + 2t\delta^T + \delta\delta^T)\right]\\ &= \mathrm{Tr}\left[G^TG(\tilde{\mathbf{Q}} - 2t\tilde{\tau}^T + tt^T)\right]\\ &\quad + \delta^TG^TG\delta + 2\delta^TG^TG(t - \tilde{\tau}).\end{aligned} \tag{30}$$

In order to ensure robustness, we aim to minimize the LS error under the worst possible sensor location errors δ.

By minimizing the maximum LS objective function, we can formulate the problem as

$$\begin{aligned}&\min_{\tilde{\mathbf{Q}},\mu} \mu\\ &\text{s.t. } \mathrm{Tr}\left[G^TG(\tilde{\mathbf{Q}} - 2t\tilde{\tau}^T + tt^T)\right]\\ &\qquad + \delta^TG^TG\delta + 2\delta^TG^TG(t - \tilde{\tau}) \le \mu,\\ &\quad \text{for all } \|\delta\| \le \epsilon\sqrt{N}/c.\end{aligned} \tag{31}$$

The constraint in (31) is equivalent to the implication relationship

$$\begin{aligned}\|\delta\| \le \epsilon\sqrt{N}/c \Rightarrow \mathrm{Tr}&\left[G^TG(\tilde{\mathbf{Q}} - 2t\tilde{\tau}^T + tt^T)\right]\\ &+\delta^TG^TG\delta + 2\delta^TG^TG(t - \tilde{\tau}) \le \mu\end{aligned} \tag{32}$$

which can be written in matrix forms as

$$\begin{aligned}&\begin{bmatrix}\delta\\1\end{bmatrix}^T\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -N\epsilon^2/c^2\end{bmatrix}\begin{bmatrix}\delta\\1\end{bmatrix} \le 0\\ &\Rightarrow \begin{bmatrix}\delta\\1\end{bmatrix}^T\begin{bmatrix}G^TG & G^TG(t-\tilde{\tau})\\ \left(G^TG(t-\tilde{\tau})\right)^T & h-\mu\end{bmatrix}\begin{bmatrix}\delta\\1\end{bmatrix} \le 0\end{aligned} \tag{33}$$

where $h = \mathrm{Tr}[G^TG(\tilde{\mathbf{Q}} - 2t\tilde{\tau}^T + tt^T)]$.

To find feasible solutions, we resort to the S-procedure in control theory [17] as in [6]. More specifically, the implication (33) holds if and only if there exists a $\lambda \ge 0$ such that

$$\begin{bmatrix}G^TG & G^TG(t-\tilde{\tau})\\ \left(G^TG(t-\tilde{\tau})\right)^T & h-\mu\end{bmatrix} \preceq \lambda\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\frac{N\epsilon^2}{c^2}\end{bmatrix}. \tag{34}$$

This convex constraint can now be added into the 2LS algorithm to improve the robustness against the sensor location error.

Additionally, similar to (17), we have the constraints for $\tilde{\mathbf{Q}}$, where

$$\begin{aligned}\tilde{\mathbf{Q}}_{ii} &= \tilde{\tau}_i^2 = \frac{1}{c^2}(\tilde{\mathbf{x}}_i - \mathbf{y})^T(\tilde{\mathbf{x}}_i - \mathbf{y})\\ &= \frac{1}{c^2}\begin{bmatrix}\tilde{\mathbf{x}}_i\\-1\end{bmatrix}^T\begin{bmatrix}\mathbf{I} & \mathbf{y}\\ \mathbf{y}^T & y_s\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{x}}_i\\-1\end{bmatrix},\end{aligned} \tag{35}$$

$$\begin{aligned}\tilde{\mathbf{Q}}_{ij} &= \tilde{\tau}_i\tilde{\tau}_j = \frac{1}{c^2}\|\tilde{\mathbf{x}}_i - \mathbf{y}\|\|\tilde{\mathbf{x}}_j - \mathbf{y}\|\\ &\ge \frac{1}{c^2}\left|\begin{bmatrix}\tilde{\mathbf{x}}_i\\-1\end{bmatrix}^T\begin{bmatrix}\mathbf{I} & \mathbf{y}\\ \mathbf{y}^T & y_s\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{x}}_j\\-1\end{bmatrix}\right|,\end{aligned} \tag{36}$$

for $1 \le i \le j \le N$, $y_s = \mathbf{y}^T\mathbf{y}$. The two equalities $\tilde{\mathbf{Q}} = \tilde{\tau}\tilde{\tau}^T$ and $y_s = \mathbf{y}^T\mathbf{y}$ can be relaxed as

$$\begin{bmatrix}\tilde{\mathbf{Q}} & \tilde{\tau}\\ \tilde{\tau}^T & 1\end{bmatrix} \succeq 0,\ \begin{bmatrix}\mathbf{I} & \mathbf{y}\\ \mathbf{y}^T & y_s\end{bmatrix} \succeq 0. \tag{37}$$

Combing all the constraints, we obtain the following SDP formulation for robust 2LS (R2LS) algorithm below

$$\begin{aligned}&\min_{\mathbf{y},y_s,\tilde{\tau},\tilde{\mathbf{Q}},\mu,\lambda} \mu + \eta\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\mathbf{Q}}_{ij}\\ &\text{s.t. } \begin{bmatrix}G^TG & G^TG(t-\tilde{\tau})\\ \left(G^TG(t-\tilde{\tau})\right)^T & h-\mu\end{bmatrix} \preceq \lambda\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\frac{N\epsilon^2}{c^2}\end{bmatrix},\\ &\quad h = \mathrm{Tr}\left[G^TG(\tilde{\mathbf{Q}} - 2t\tilde{\tau}^T + tt^T)\right],\\ &\quad \begin{bmatrix}\tilde{\mathbf{Q}} & \tilde{\tau}\\ \tilde{\tau}^T & 1\end{bmatrix} \succeq 0,\ \begin{bmatrix}\mathbf{I} & \mathbf{y}\\ \mathbf{y}^T & y_s\end{bmatrix} \succeq 0,\\ &\quad \tilde{\mathbf{Q}}_{ii} = \frac{1}{c^2}\begin{bmatrix}\tilde{\mathbf{x}}_i\\-1\end{bmatrix}^T\begin{bmatrix}\mathbf{I} & \mathbf{y}\\ \mathbf{y}^T & y_s\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{x}}_i\\-1\end{bmatrix},\\ &\quad \tilde{\mathbf{Q}}_{ij} \ge \frac{1}{c^2}\left|\begin{bmatrix}\tilde{\mathbf{x}}_i\\-1\end{bmatrix}^T\begin{bmatrix}\mathbf{I} & \mathbf{y}\\ \mathbf{y}^T & y_s\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{x}}_j\\-1\end{bmatrix}\right|,\\ &\quad \lambda \ge 0, \mu \ge 0.\end{aligned} \tag{38}$$

Using the SDP solver Sedumi [16], we can get the source location based on this R2LS algorithm.

C. Robust Min-Max Algorithm for Localization

We now develop a robust min-max algorithm (RMMA) for source location under sensor (anchor) location errors. We can extend the min-max formulation to the inaccurate anchor node position case by incorporating the additional location uncertainty constraints.

More specifically, we obtain the following formulation:

$$\min_{\mathbf{y},t_0}\max_{\|\xi_i\|\le\epsilon,i}\left|(t_i - t_0)^2 - \frac{1}{c^2}\|\tilde{\mathbf{x}}_i - \xi_i - \mathbf{y}\|^2\right|. \tag{39}$$

Similarly to the development of MMA, we introduce two auxiliary variables $y_s = \mathbf{y}^T\mathbf{y}$ and $t_s = t_0^2$, and rewrite (39) into

$$\begin{aligned}\min_{\mathbf{y},y_s,t_0,t_s}\max_{\|\xi_i\|\le\epsilon,i}&\left|t_s - 2t_it_0 + t_i^2 - \frac{1}{c^2}\left(y_s - 2\tilde{\mathbf{x}}_i^T\mathbf{y} + \tilde{\mathbf{x}}_i^T\tilde{\mathbf{x}}_i\right)\right.\\ &\left.-\frac{1}{c^2}\left(2\xi_i^T(\mathbf{y} - \tilde{\mathbf{x}}_i) + \xi_i^T\xi_i\right)\right|.\end{aligned} \tag{40}$$

Then, we have the following optimization problem to solve:

$$\begin{aligned}&\min_{\mathbf{y},y_s,t_0,t_s} \theta\\ &\text{s.t. } -\theta \le t_s - 2t_it_0 + t_i^2 - \frac{1}{c^2}\left(y_s - 2\mathbf{x}_i^T\mathbf{y} + \mathbf{x}_i^T\mathbf{x}_i\right)\\ &\qquad - \frac{1}{c^2}\left(2\xi_i^T(\mathbf{y} - \tilde{\mathbf{x}}_i) + \xi_i^T\delta_i\right) \le \theta,\\ &\quad \text{for all } \|\xi_i\| \le \epsilon,\ i = 1, \cdots, N.\end{aligned} \tag{41}$$

We now derive the necessary constraints to develop a convex optimization algorithm. Let

$$b_i = t_s - 2t_it_0 + t_i^2 - \frac{1}{c^2}\left(y_s - 2\tilde{\mathbf{x}}_i^T\mathbf{y} + \tilde{\mathbf{x}}_i^T\tilde{\mathbf{x}}_i\right),$$

the constraints in (41) are equivalent to the two implication relationships:

$$\|\xi_i\| \le \epsilon \Rightarrow b_i - \frac{1}{c^2}\left(2\xi_i^T(\mathbf{y}-\tilde{\mathbf{x}}_i)+\xi_i^T\xi_i\right) \ge -\theta$$
$$\|\xi_i\| \le \epsilon \Rightarrow b_i - \frac{1}{c^2}\left(2\xi_i^T(\mathbf{y}-\tilde{\mathbf{x}}_i)+\xi_i^T\xi_i\right) \le \theta. \quad (42)$$

Furthermore, we can express the implications in matrix form, where

$$\begin{bmatrix}\xi_i\\1\end{bmatrix}^T\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\epsilon^2\end{bmatrix}\begin{bmatrix}\xi_i\\1\end{bmatrix} \le 0$$
$$\Rightarrow \begin{bmatrix}\xi_i\\1\end{bmatrix}^T\begin{bmatrix}\mathbf{I} & \tilde{\mathbf{x}}_i-\mathbf{y}\\ (\tilde{\mathbf{x}}_i-\mathbf{y})^T & -c^2(b_i+\theta)\end{bmatrix}\begin{bmatrix}\xi_i\\1\end{bmatrix} \le 0$$
$$\begin{bmatrix}\xi_i\\1\end{bmatrix}^T\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\epsilon^2\end{bmatrix}\begin{bmatrix}\xi_i\\1\end{bmatrix} \le 0$$
$$\Rightarrow \begin{bmatrix}\xi_i\\1\end{bmatrix}^T\begin{bmatrix}-\mathbf{I} & -(\tilde{\mathbf{x}}_i-\mathbf{y})\\ -(\tilde{\mathbf{x}}_i-\mathbf{y})^T & c^2(b_i-\theta)\end{bmatrix}\begin{bmatrix}\xi_i\\1\end{bmatrix} \le 0. \quad (43)$$

Based on the S-procedure mentioned earlier, the implications hold if and only if there exist $\alpha_i \ge 0$ and $\beta_i \ge 0$ such that

$$\begin{bmatrix}\mathbf{I} & \tilde{\mathbf{x}}_i-\mathbf{y}\\ (\tilde{\mathbf{x}}_i-\mathbf{y})^T & -c^2(b_i+\theta)\end{bmatrix} \preceq \alpha_i\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\epsilon^2\end{bmatrix}$$
$$\begin{bmatrix}-\mathbf{I} & -(\tilde{\mathbf{x}}_i-\mathbf{y})\\ -(\tilde{\mathbf{x}}_i-\mathbf{y})^T & c^2(b_i-\theta)\end{bmatrix} \preceq \beta_i\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\epsilon^2\end{bmatrix}. \quad (44)$$

Thus, we now have convex inequalities in (44) to be incorporated into the original MMA for more robust location estimates.

As in the development of the original MMA, the two equalities $y_s = \mathbf{y}^T\mathbf{y}$ and $t_s = t_0^2$ can be relaxed into inequalities $y_s \succeq \mathbf{y}^T\mathbf{y}$ and $t_s \succeq t_0^2$, respectively. By expressing them in terms of linear matrix inequalities, we now have convex constraints

$$\begin{bmatrix}\mathbf{I} & \mathbf{y}\\ \mathbf{y}^T & y_s\end{bmatrix} \succeq 0,\ \begin{bmatrix}1 & t_0\\ t_0 & t_s\end{bmatrix} \succeq 0. \quad (45)$$

Combining the preceding convex constraints, we arrive at an RMMA in a SDP formulation

$$\begin{aligned}&\min_{\mathbf{y},y_s,t_0,t_s,b_i,\alpha_i,\beta_i} \theta\\
&\text{s.t. } \begin{bmatrix}\mathbf{I} & \mathbf{y}\\ \mathbf{y}^T & y_s\end{bmatrix} \succeq 0,\ \begin{bmatrix}1 & t_0\\ t_0 & t_s\end{bmatrix} \succeq 0,\\
&b_i = t_s - 2t_it_0 + t_i^2 - \frac{1}{c^2}\left(y_s - 2\tilde{\mathbf{x}}_i^T\mathbf{y} + \tilde{\mathbf{x}}_i^T\tilde{\mathbf{x}}_i\right),\\
&\begin{bmatrix}\mathbf{I} & \tilde{\mathbf{x}}_i-\mathbf{y}\\ (\tilde{\mathbf{x}}_i-\mathbf{y})^T & -c^2(b_i+\theta)\end{bmatrix} \preceq \alpha_i\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\epsilon^2\end{bmatrix},\\
&\begin{bmatrix}-\mathbf{I} & -(\tilde{\mathbf{x}}_i-\mathbf{y})\\ -(\tilde{\mathbf{x}}_i-\mathbf{y})^T & c^2(b_i-\theta)\end{bmatrix} \preceq \beta_i\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\epsilon^2\end{bmatrix},\\
&\qquad \alpha_i \ge 0, \beta_i \ge 0,\\
&\qquad i = 1,\cdots,N.\end{aligned} \quad (46)$$

As mentioned before, the RMMA can also be solved via interior point methods.

D. CRLB Under Sensor Node Location Errors

We would like to analyze the effect of node location error on the performance limit of source localization. Similar to [18], [19], we integrate the sensor location errors when deriving the Fisher information matrix.

Let the TOA measurement noise n_i be i.i.d. Gaussian with zero mean and variance σ^2 and let the anchor node location errors ξ_i also be i.i.d. Gaussian with zero mean and variance $\sigma_x^2 I$. Under these assumptions, the measurement data$t_1, t_2, \ldots, t_N$, and $\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \ldots, \tilde{\mathbf{x}}_N$ follow a joint Gaussian distribution

$$\begin{aligned}&p\left(t_1,\ldots,t_N,\tilde{\mathbf{x}}_1,\ldots,\tilde{\mathbf{x}}_N|\mathbf{y},\mathbf{x}_1,\ldots,\mathbf{x}_N,t_0\right)\\
&\quad= (2\pi\sigma^2)^{-\frac{N}{2}}\exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^N\left(t_i - \frac{1}{c}\|\mathbf{x}_i-\mathbf{y}\| - t_0\right)^2\right)\\
&\quad\times\left(2\pi\sigma_x^2\right)^{-\frac{N}{2}}\exp\left(-\frac{1}{2\sigma_x^2}\sum_{i=1}^N\|\tilde{\mathbf{x}}_i-\mathbf{x}_i\|^2\right).\end{aligned} \quad (47)$$

Define $\mathbf{W} = (\mathbf{y}^T, \mathbf{x}_1^T, \ldots \mathbf{x}_N^T, t_0)$. The log-likelihood function of the location estimation (ignoring the constant term) is given by

$$L(\mathbf{W}) = -\frac{1}{2\sigma^2}\sum_{i=1}^N\left(t_i - \frac{1}{c}\|\mathbf{x}_i-\mathbf{y}\| - t_0\right)^2 - \frac{1}{2\sigma_x^2}\sum_{i=1}^N\|\tilde{\mathbf{x}}_i-\mathbf{x}_i\|^2. \quad (48)$$

From (48), we can determine the (k,j)th element of the Fisher information matrix $\mathbf{F_c}$ as

$$\begin{aligned}[\mathbf{F_c}]_{kl} &= -E\left(\frac{\partial^2}{\partial w_k\partial w_l}L(\mathbf{W})\right)\\
&= \frac{1}{c^2\sigma^2}\sum_{i=1}^N\frac{(y_k-x_{ik})(y_j-x_{il})}{\|\mathbf{x}_i-\mathbf{y}\|^2},\\
&\quad 1 \le k \le l \le m\end{aligned} \quad (49)$$

$$\begin{aligned}[\mathbf{F_c}]_{kk} &= -E\left(\frac{\partial^2}{\partial^2 w_k}L(\mathbf{W})\right) = \frac{1}{\sigma_x^2},\\
&\quad m+1 \le k \le (N+1)m;\end{aligned} \quad (50)$$

$$\begin{aligned}[\mathbf{F_c}]_{kl} &= [\mathbf{F_c}]_{lk} = -E\left(\frac{\partial^2}{\partial w_k\partial w_l}L(\mathbf{W})\right) = 0,\\
&\quad m+1 \le k < l \le (N+1)m.\end{aligned} \quad (51)$$

Additionally, we have, for $1 \le k \le m$,

$$\begin{aligned}[\mathbf{F_c}]_{k(Nm+m+1)} &= [\mathbf{F_c}]_{(Nm+m+1)k}\\
&= -E\left(\frac{\partial^2}{\partial w_k\partial w_{Nm+m+1}}L(\mathbf{W})\right)\\
&= \frac{1}{c\sigma^2}\sum_{i=1}^N\frac{y_k-x_{ik}}{\|\mathbf{x}_i-\mathbf{y}\|}\end{aligned} \quad (52)$$

and

$$[\mathbf{F_c}]_{(Nm+m+1)(Nm+m+1)} = -E\left(\frac{\partial^2}{\partial^2 w_{Nm+m+1}}L(\mathbf{W})\right) = \frac{N}{\sigma^2}. \quad (53)$$

Hence, the CRLB of the unbiased estimate $\hat{\mathbf{y}}$ is

$$\text{MSE}_c = E\left(\|\hat{\mathbf{y}}-\mathbf{y}\|^2\right) \ge \sum_{i=1}^m\left[\mathbf{F_c^{-1}}\right]_{ii}. \quad (54)$$

TABLE I
COMPLEXITY COMPARISON OF DIFFERENT ALGORITHMS

Algorithm	Iteration Number	Operation Per Iteration
Classic-TDOA	1	$O(mN)$
SDP-TDOA	$O((2m)^{1/2})$	$O((N^2+N+m+1)^2N^2)$
2LS	$O((2m)^{1/2})$	$O((N^2+N+m+1)^2N^2)$
MMA	$O((m+1)^{1/2})$	$O((m+2)^2N)$
R2LS	$O((2m)^{1/2})$	$O((N^2+N+m+3)^2N^2)$
RMMA	$O((m+1)^{1/2})$	$O((2N+m+2)^2N)$

Despite the fact that no efficient unbiased estimate exists for the TOA measurement model, the CRLB can still provide a reasonable benchmark.

V. Complexity Comparisons

We compare the complexity of different algorithms discussed thus far. We apply the result of [20] to evaluate the complexity of various convex algorithms. In Table I, we summarize algorithm complexities in terms of the number of iterations and operations in each iteration. We label the classic TDOA algorithm in [2] as "Classic-TDOA", the SDP TDOA algorithm in [6] as "SDP-TDOA".

From this comparison, we can see that the complexity of the proposed new TOA algorithms are higher than the classic TDOA algorithms. This is the tradeoff for more reliable convergence performance. The 2LS approach and the SDP-TDOA approach have similar complexity due to similar semidefinite relaxation. Since the 2LS and the R2LS algorithms require more slack variables to optimize, they also have higher complexity than the MMA and the RMMA algorithms. The complexity difference is more pronounced in terms of the operations in each iteration particularly when N (the number of sensors) is large.

VI. Simulation Results

A. Simulation Setup

In this section, we provide several test examples to demonstrate the performance of the proposed TOA algorithms and in comparison with the classic TDOA algorithm [2] (labeled as Classic-TDOA) and the SDP TDOA algorithm [6] (labeled as SDP-TDOA). Since the noise covariance matrix of the SDP-TDOA algorithm is not invertible if we include all the pairwise TDOA measurements, we only select one anchor node as the reference node and utilize the corresponding TDOA measurement for estimation.

We note that some localization works in literature add a local refinement search step after finding an approximate solution to improve the overall performance [6], [9], [21]. Equivalently, this implies a two step procedure: a) convex optimization solution for an initial estimate; b) a local refinement to minimize the nonlinear LS criterion (eq. (2)) based on the initial point from step a). Most of the time, however, we see that the gradient search for the optimum LS solution (2) using Powell algorithm [22] provides a final convergence point near the true source location. Typically, the closer the initial point is to the true location, the faster the local search will converge to the final solution, and the less likely it will be trapped in the local minimum.

Note that there is no established standard for performance comparison. Thus, two ways of comparison can be made. One is to compare the performance of different algorithms after step a), and the other is to compare the performance after step b). However, if we *only* demonstrate the localization results after the additional local search step b), the results would obscure the effect of the different algorithms in step a). In fact, without results from step a), the final convergence would be misleading as it is difficult to differentiate the residual error of different algorithms. This is because when using the same local search criterion (2), the search results often converge to the same point; such is the case that we have observed for the examples we tested. Therefore, in order to make a fair comparison of different algorithms, we present the performance of different algorithms without additional local search in our paper. We can then show the true result of different optimization procedure. As a result, comparative results from purely the optimization step are more illustrative of their efficacy.

In all the results we will show, additional local search step b) will also be implemented by using Powell algorithm for implementing the OLS. Hence, the OLS results represent the final convergence of various comparative algorithms after local search step b).

We remark on the implication of the comparison after step a). The error surface defined by (2) is quite complex, depending on the locations of the sensors and the source. Thus, there is no guarantee that smaller localization error after the optimization algorithm necessarily leads to faster and more accurate convergence for the local search. Nevertheless, based on some known results [9], [21] with respect to the existence of local minimum for localization problems, we expect that the closer the "raw" result is to the true location, the faster the local search will converge to the final solution, and the less likely it will be trapped in local minimum.

In our test, we place eight sensors in a 2-dimensional area at $\mathbf{x}_1 = [400, 400]^T$, $\mathbf{x}_2 = [400, -400]^T$, $\mathbf{x}_3 = [-400, 400]^T$, $\mathbf{x}_4 = [-400, -400]^T$, $\mathbf{x}_5 = [800, 800]^T$, $\mathbf{x}_6 = [800, -800]^T$, $\mathbf{x}_7 = [-800, 800]^T$, $\mathbf{x}_8 = [-800, -800]^T$. We evaluate the root mean-square error (RMSE) of the source position as the performance metric against different strengths of the noise standard deviation. For simplicity, we convert the noise into the distance domain.

In the numerical results, we include both the CRLB for the TOA model derived in Section III and the CRLB of the TDOA model which can be found in [14]. In the figures, these bounds are labeled as CRLB-TDOA and CRLB-TOA, respectively.

B. Monte Carlo Simulations

Example 1: In this example, the source is placed at point $[30, 10]^T$, which is inside the convex hull formed by the sensor/anchor nodes. The noise is generated as i.i.d. Gaussian, and t_0 is randomly chosen with normal distribution of zero mean and variance of 4. The penalty factor α is set to 6.18×10^{-5} for the 2LS algorithm. We consider no sensor location errors. In Fig. 1, we compare the performance of Classic-TDOA, SDP-TDOA, 2LS, MMA, and OLS algorithms. It can be seen that the performance of the proposed new TOA algorithms are better than Classic-TDOA and SDP-TDOA approach. The performance of 2LS and OLS algorithms are very close to each other and also close to the CRLB of the TDOA model. This means that the 2LS

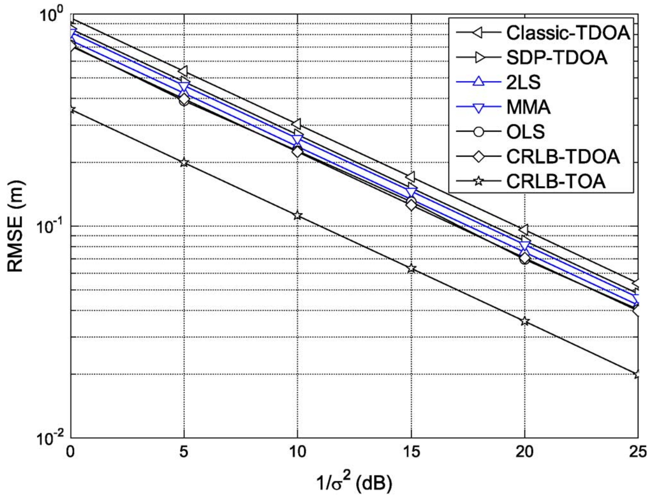


Fig. 1. Comparison of Classic-TDOA, SDP-TDOA, 2LS, MMA and OLS algorithms when a source node is inside the convex hull, Gaussian noise.

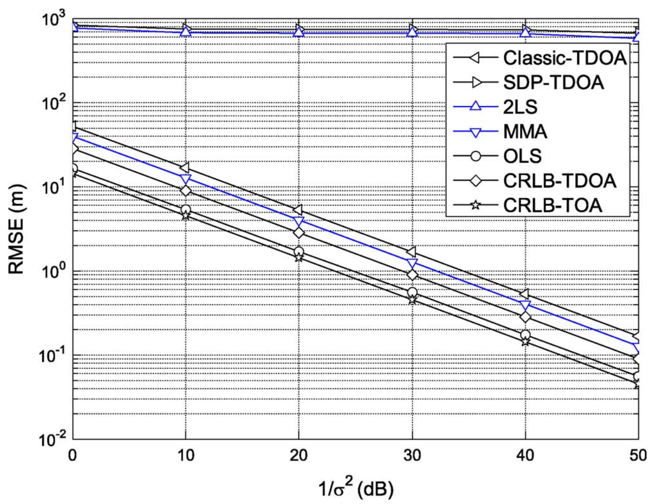


Fig. 2. Comparison of Classic-TDOA, SDP-TDOA, 2LS, MMA and OLS algorithms when a source node is outside the convex hull, Gaussian noise.

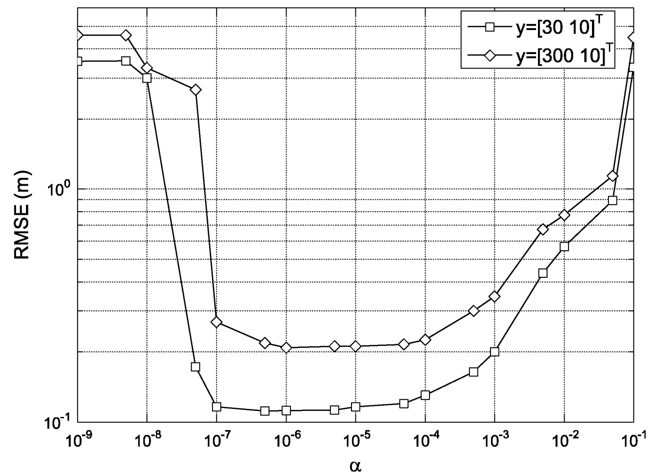


Fig. 3. Selection of the penalty factor α in 2LS.

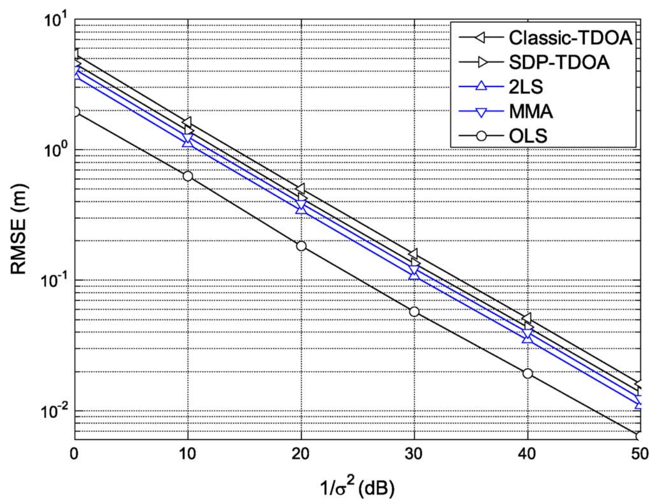


Fig. 4. Comparison of Classic-TDOA, SDP-TDOA, 2LS, MMA and OLS algorithms when a source node is uniformly distributed in a square region, Gaussian noise.

algorithm can achieve performance very close to the CRLB if pairwise TOA subtractions are utilized to generate the TDOA measurement data.

Example 2: In this example, We position the source node at $[3000, 10]^T$, which is now outside the convex hull of the sensor nodes. We set t_0 to be normally distributed with zero mean and variance of 4 and i.i.d. Gaussian measurement noise. The parameter α for 2LS algorithm is set to 6.18×10^{-5}. The performance of the various algorithms along with the bounds are given in Fig. 2.

From the results, we can see that the OLS provides the best performance. In fact, because of the 3-dB SNR loss in the TDOA model, OLS generated performance better than the CRLB under TDOA. The MMA approach is better than the Classic-TDOA and is about 2 dB away from the CRLB of the TDOA model. Unfortunately, both SDP-TDOA and 2LS fail to give a good estimation in this case. One reason for this is that the source node is far away and the SDP optimization is unable to escape a local minimum. It is also interesting to note that the OLS algorithm out performs the CRLB derived for the TDOA model. This comparison demonstrates the drawback of preprocessing that led to the TDOA model. Moreover, we also observe that a significant gap exists between the CRLB-TOA and all the tested algorithms. This observation illustrates that there may still exist room for potentially significant improvement of source localization in the TOA model. Thus, we should continue the development of new and better algorithms to improve the source localization accuracy based on TOA measurement.

Example 3: Here we test the sensitivity of the proposed 2LS algorithm to the selection of the penalty factor. We fix the Gaussian noise variance σ^2 to 15 dB and the source position to $[30, 10]^T$ and $[300, 10]^T$. The RMSE values of $\mathbf{y}$ by applying different values of α is compared in Fig. 3. It can be seen that the algorithm is not very sensitive to the choice of the penalty factor α. Our experience shows that α can be chosen between 10^{-6} and 10^{-4} for reliable estimation of $\mathbf{y}$.

Example 4: We place the source node in the square region: $\{(a,b)| -1200 \le a \le 1200, -1200 \le b \le 1200\}$. For each σ, we randomly generate 3000 locations uniformly. The noise is i.i.d. Gaussian and $\alpha = 6.18 \times 10^{-5}$ for the 2LS algorithm. t_0 is randomly chosen with normal distribution of zero mean and variance of 4. In Fig. 4, we show the performance

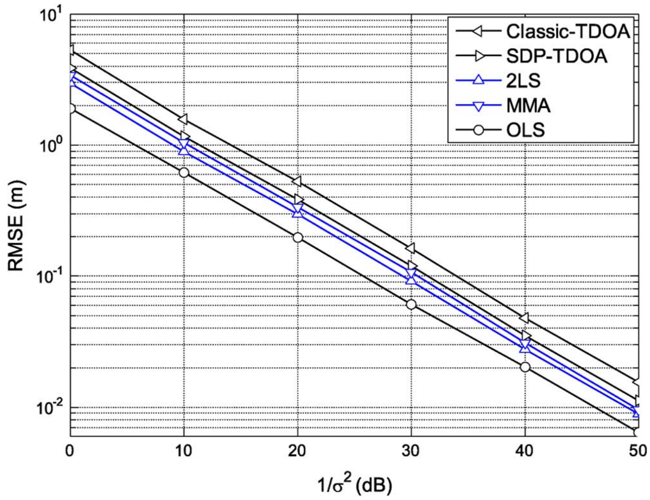


Fig. 5. Comparison of Classic-TDOA, SDP-TDOA, 2LS, MMA and OLS algorithms when a source node is uniformly distributed in a square region, uniformly distributed noise.

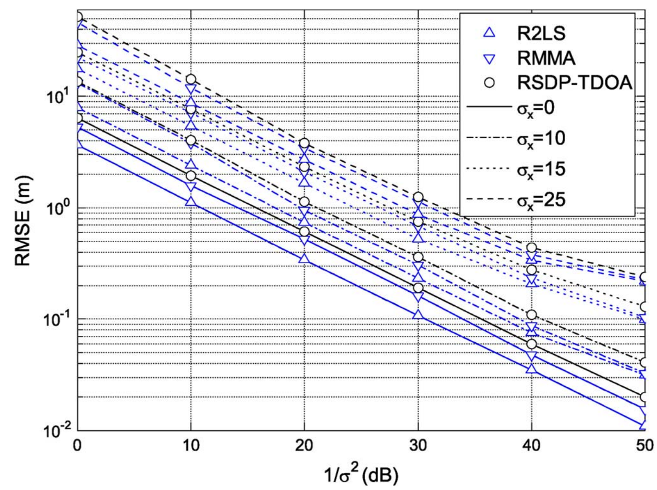


Fig. 6. Comparison of R2LS, RMMA and RSDP-TDOA algorithm with accurate and inaccurate anchor node locations, Gaussian noise.

of different algorithms. The results show that when the source is at different positions, the brute force OLS approach gives the best performance. The 2LS and the MMA significantly outperform the Classic-TDOA approach by moving closer to the global minimum.

Example 5: In this example, we also position the source node in the square region: $\{(a,b)| -1200 \leq a \leq 1200, -1200 \leq b \leq 1200\}$ like in Example 4. We now consider uniformly distributed noise in this case and test the performance of different algorithms in this example. The signal transmission time t_0 is normally distributed with zero mean and variance of 4. The penalty factor α is set to 6.18×10^{-5} for the 2LS algorithm. From Fig. 5, we can find that when the noise is uniformly distributed, our proposed two algorithms are still robust and continue to work well. Both of them are better than the SDP-TDOA approach, and offer about 4 dB gain over the Classic-TDOA algorithm, and are within 3 dB from the OLS approach.

Example 6: We consider, in this example, the effect of sensor location error. We place the source node in the square region: $\{(a,b)| -1200 \leq a \leq 1200, -1200 \leq b \leq 1200\}$ and use Gaussian model for the noise of in the TOA measurement. The penalty factor α of the R2LS algorithm is set to 6.18×10^{-5}. We compare the performance of our R2LS and RMMA algorithm with the robust SDP-TDOA algorithm (denoted by RSDP-TDOA) under inaccurate sensor location in Fig. 6. Notice that we let σ_x denote the location error variance of the anchor or sensor nodes. By default, $\sigma_x = 0$ represents the case involving only accurate sensor locations. The simulation results in Fig. 6 show that when the sensor node location errors are modest to relatively low $(\sigma_x \leq 10)$, the proposed R2LS and RMMA can still obtain good estimates, and are better than the robust SDP-TDOA approach. When the sensor node location errors are significant, however, the performance loss is relatively substantial. In this case, the errors in the sensor node locations are so large that our original approximation neglecting high order terms in the sensor location uncertainty model simply does not hold. We also note that since the RMMA method involves more optimization variables, its performance is slightly worse than R2LS under sensor location uncertainty.

C. Summary

From the simulation results, we find that our proposed two algorithms provide a better estimate compared with Classical-TDOA and SDP-TDOA approaches. The 2LS algorithm has a similar formulation as the SDP-TDOA approach. However, the 2LS approach utilizes N measurements and whereas the SDP-TDOA algorithm uses $N-1$ measurements, and they adopt different objective functions. As a result, the performance of the 2LS approach is better. The MMA approach is less sensitive to the relative location of the source node to the sensors, and performs well in all cases. When there are location errors for the sensors, the proposed R2LS and RMMA methods can still give a good estimation.

VII. Conclusion

We investigate the problem of source localization in wireless sensor networks based on the practical TOA model. Directly taking the TOA measurement, our study is less susceptible to the 3 dB noise enhancement and does not require prior knowledge on the signal transmission time. We develop two convex optimization methods for direct source localization using SDP. We also propose means to obtain robust location estimation when sensor node locations are subject to errors. Our results demonstrate the performance advantage of the newly developed TOA algorithm for source localization over the traditionally used TDOA preprocessing, under various noise conditions and in the presence of sensor location errors.

Acknowledgment

The authors would like to thank the editor and several anonymous reviewers for their helpful suggestions and comments.

References

[1] N. Patwari, J. Ash, S. Kyperountas, A. Hero, R. Moses, and N. Correal, "Locating the nodes: Cooperative localization in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 54–69, Jul. 2005.

[2] A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: Challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 24–40, Jul. 2005.

[3] N. Patwari, A. Hero, M. Perkins, N. Correal, and R. O'Dea, "Relative location estimation in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 1, no. 8, pp. 2137–2148, Aug. 2003.

[4] K. W. Cheung, H. C. So, W. K. Ma, and Y. T. Chan, "Least squares algorithms for time-of-arrival-based mobile location," *IEEE Trans. Signal Process.*, vol. 52, no. 4, pp. 1121–1128, Apr. 2004.

[5] K. C. Ho and Y. T. Chan, "Solution and performance analysis of geolocation by TDOA," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 29, no. 4, pp. 1311–1322, Oct. 1993.

[6] K. Yang, G. Wang, and Z. Luo, "Efficient convex relaxation methods for robust target localization by a sensor network using time differences of arrivals," *IEEE Trans. Signal Process.*, vol. 57, no. 7, pp. 2775–2784, Jul. 2009.

[7] K. W. K. Lui, F. K. W. Chan, and H. C. So, "Semidefinite programming approach for range-difference based source localization," *IEEE Trans. Signal Process.*, vol. 57, no. 4, pp. 1631–1633, Apr. 2009.

[8] P. Tseng, "Second-order cone programming relaxation of sensor network localization," *SIAM J. Optim.*, vol. 18, pp. 156–185, Jan. 2008.

[9] P. Biswas, T. Lian, T. Wang, and Y. Ye, "Semidefinite programming based algorithms for sensor network localization," *ACM Trans. Sensor Netw.*, vol. 2, no. 2, pp. 188–220, May 2006.

[10] K. W. Cheung, W. K. Ma, and H. C. So, "Accurate approximation algorithm for TOA-based maximum likelihood mobile location using semidefinite programming," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Mar. 2004, vol. 2, pp. 145–148.

[11] J. Moré and Z. Wu, "Global continuation for distance geometry problems," *SIAM J. Optim.*, vol. 7, pp. 814–836, 1997.

[12] Y. Qi, H. Kobayashi, and H. Suda, "Analysis of wireless geolocation in a non-line-of-sight environment," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 672–681, Mar. 2006.

[13] A. Host-Madsen, "On the existence of efficient estimators," *IEEE Trans. Signal Process.*, vol. 48, no. 11, pp. 3028–3031, Nov. 2000.

[14] E. Y. Xu, Z. Ding, and S. Dasgupta, "Wireless source localization based on time of arrival measurement," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Mar. 2010, pp. 2842–2845.

[15] S. Boyd and L. Vandenberghe*, Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[16] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optim. Methods Softw.* vol. 11–12, pp. 625–653, 1999 [Online]. Available: http://sedumi.mcmaster.ca

[17] V. Yakubovich, "S-procedure in nonlinear control theory," *Vestnik Leningrad University*, vol. 1, pp. 62–77, 1971.

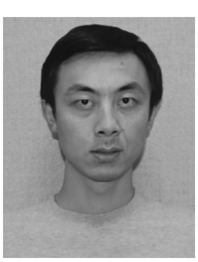
[18] Y. Rockah, H. Messer, and P. M. Schultheiss, "Localization performance of arrays subject to phase errors," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 24, no. 4, pp. 402–410, Jul. 1988.

[19] N. H. Lehmann, A. M. Haimovich, R. S. Blum, and L. Cimini, "High resolution capabilities of MIMO radar," *40th Asilomar Conf. Signals, Syst., Comput. (ACSSC)*, pp. 25–30, 2007.

[20] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Rev.*, vol. 38, no. 1, pp. 49–95, Mar. 1996.

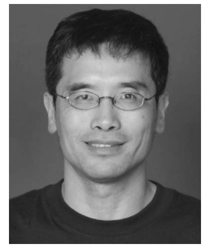
[21] B. Fidan, S. Dasgupta, and B. Anderson, "Guaranteeing practical convergence in algorithms for sensor and source localization," *IEEE Trans. Signal Process.*, vol. 56, no. 9, pp. 4458–4469, Sep. 2008.

[22] J. Nocedal and S. J. Wright*, Numerical optimization*. New York: Springer-Verlag, 1999.



Enyang Xu (S'09) received the B.E. and M.S. degrees from the National Mobile Communications Research Laboratory, Southeast University, Nanjing, China, in 2005 and 2008, respectively.

He is currently working towards the Ph.D. degree in the Department of Electrical and Computer Engineering at the University of California, Davis. His research interests are in the area of wireless communications and signal processing, with current emphasis on source localization, tracking, joint detection, and decoding.



Zhi Ding (S'88–M'90–SM'95–F'03) received the Ph.D. degree in electrical engineering from Cornell University, Ithaca, NY, in 1990.

From 1990 to 1998, he was a faculty member of Electrical and Computer Engineering at the University of Iowa. He joined the University of California, Davis, in 2000 as a Professor of Electrical and Computer Engineering. He is currently the Child Family Endowed Professor of Engineering and Entrepreneurship at the University of California, Davis. He is also a guest Changjiang Chair Professor of the Southeast University, Nanjing, China. He has published over 200 refereed research papers. He also coauthored two books: *Blind Equalization and Identification* (Taylor and Francis, 2001) and *Modern Digital and Analog Communication Systems* (Oxford Univ. Press, 2009). His major research interests are wireless communications, networking, and signal processing.

Dr. Ding was named as 2004–2006 Distinguished Lecturer by the Circuits and Systems Society. He was Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 1994 to 1997 and from 2001 to 2004. He served as Associate Editor for the IEEE SIGNAL PROCESSING LETTERS from 2002 to 2005. He was a member of the Editorial Board of the *IEEE Signal Processing Magazine*. He served as the Technical Program Chair for COMSOCs GLOBECOM 2006. He was a member of the IEEE Signal Processing Society Technical Committee on Statistical Signal and Array Processing from 1994 to 1998 and a member of Technical Committee on Signal Processing for Communications from 1998 to 2003. He also served on the IEEE Signal Processing Society Technical Committee on Multi-Media Signal Processing from 2002 to 2005. He was a Distinguished Lecturer of the IEEE Communications Society for the term 2008–2009.

Soura Dasgupta (M'87–SM'93–F'98) received the B.E. degree in electrical engineering from the University of Queensland, Australia, in 1980 and the Ph.D. degree in systems engineering from the Australian National University in 1985.

In 1981, he was a Junior Research Fellow in the Electronics and Communications Sciences Unit at the Indian Statistical Institute, Calcutta, India. He is currently Professor of Electrical and Computer Engineering at the University of Iowa, Iowa City. He has held visiting appointments at the University of Notre Dame, the University of Iowa, the Université Catholique de Louvain-La-Neuve, Belgium, and the Australian National University. His research interests are in controls, signal processing and communications.

Dr. Dasgupta served, respectively, between 1988 and 1991, 1998 to 2009, and 2004 and 2007, as an Associate Editor of the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, the IEEE Control Systems Society Conference Editorial Board, and the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS II. He is a corecipient of the Gullimen Cauer Award for the best paper published in the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS in 1990 and 1991, a past Presidential Faculty Fellow, a subject editor for the *International Journal of Adaptive Control and Signal Processing*, and a member of the editorial board of the *EURASIP Journal of Wireless Communications*.